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Exchange rate volatility and cooperation in an incomplete markets' economy

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Abstract

In this paper, we contribute to the debate on whether exchange rate volatility is detrimental or not for welfare by characterizing optimal monetary policies in a two-country OLG model where markets are incomplete. The equilibrium nominal exchange rate is volatile as a result of shocks against which agents are not able to insure. In a non-cooperative environment, central banks have an incentive to devaluate the domestic currency by giving monetary transfers to domestic agents. However, such policies result in higher exchange rate volatility. We show that cooperation reduces exchange rate volatility as: (1) the negative spillover effects due to the expansionary monetary policies are internalized; (2) cooperative policies smooth the effects of shocks to fundamentals on the exchange rate. For standard parameter values, the gains from cooperation are not negligible. However, for cooperation to be Pareto improving countries should be weighted differently in the social welfare function. This could explain the lack of cooperation across countries, instead of the negligible gains as previously argued.

Keywords: exchange rate volatility, incomplete markets, international spillovers, gains from cooperation, OLG models

JEL classification: D52; F31; F41; F42

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1 Introduction

The question of whether exchange rate volatility is detrimental or not from a welfare point of view is long-standing in economics. Although many countries have officially adopted a flexible exchange rate regime since the collapse of the Bretton Woods system, many central banks intervene in foreign exchange markets as they seem to have a sense of unease about letting exchange rates be solely determined by market fundamentals. In fact, only 15.2% of the IMF members let their currency freely float and the number of countries with a *de facto* free floating exchange rate arrangement is decreasing[1]. There is also an increasing perception that we are living in an era of "currency wars", where countries use monetary policy to manipulate the exchange rate at their advantage. It is often thought that one of the side effects of these "uncoordinated actions" can be too much exchange rate volatility[2]. In this context, calls for international cooperation are becoming increasingly common[3].

The aim of this paper is to revisit the question of whether exchange rate volatility should be a concern for monetary policy in a model where markets are incomplete. The main idea behind the paper can be briefly described as follows. When markets are incomplete, it is well known that the competitive equilibrium is not Pareto optimal (e.g. Geanakoplos and Polemarchakis, 1986). If the nominal exchange rate is volatile as a consequence of shocks to fundamentals of the economy, against which agents are not able to insure, then it follows that the exchange rate fluctuations which arise in the economy are inefficient. In this context, central banks' attempts to influence the nominal exchange rate might have a rationale. We then ask the following questions. Firstly, what is the impact of monetary policy on the nominal exchange rate when central banks set their policies optimally but act in a non-cooperative environment? Secondly, does cooperation among central banks imply that exchange rate volatility is reduced as a result or not? Finally, what are the welfare consequences of cooperation and are there sizable gains?

To answer these questions, we consider a tractable two-country stochastic OLG model. Assuming that the utility function is logarithmic in consumption, we are able to find closed-form solutions for the competitive equilibrium and for optimal policy rules. With log utility, it is well known markets are effectively complete with productivity shocks alone

[1] See IMF (2014).

[2] See e.g. Portes (2012).

[3] See Blanchard (2016) and Frankel (2016) for recent discussions.

(Cole and Obstfeld, 1991). In our framework, market incompleteness will then result from agents' inability to insure against demand shocks. In particular, agents gain utility from the domestic and the foreign good (as well as leisure) but the weights assigned to the two goods in the utility function depend on the state of nature in which the agent is born[4].

In the first period, agents make a decision about how much labour to supply to domestic firms, which produce a country-specific good, while in the second period they consume. The saving instruments are two currencies, which serve as stores of value as it is standard in overlapping-generations models with money. Hence, agents make a portfolio decision between the domestic and the foreign currency. After the shock takes place in the following period, the domestic (foreign) currency is then used to buy the domestic (foreign) good. Central banks can print money supply, hence their monetary policy instrument is the rate of money growth[5]. For the sake of realism, the (nominal) monetary transfers can only be given to domestic agents. In equilibrium, central banks are able to affect the dynamics of the nominal exchange rate by changing the monetary transfers. Our object of analysis will be the stationary equilibrium of the economy, which is defined as a distribution across states of nature of the consumption allocation[6].

After proving that a stationary equilibrium exists and is unique, we set up a planner's problem where the objective is to maximize a weighted sum of the utility of all agents in the world economy subject to the feasibility constraints and the production functions. Agents are distinguished not only by their country, but also by the state of nature in which they are born. The notion of optimality that we use is Conditional Pareto Optimality (CPO), which requires that, conditional on being born in a particular state of nature, there is no other allocation which makes the agents better off. We then prove that the competitive equilibrium is not CPO. This means that markets are sequentially incomplete i.e. agents are not able to insure against the risk they face when old[7]. In equilibrium, the exchange rate is volatile as a consequence of the uncertainty on the fundamentals of the economy, i.e. agents' preferences for the domestic and the foreign good. Since the

[4]Pavlova and Rigobon (2008, 2010) also focus on shocks to the degree of home bias to study the dynamics of asset prices and portfolios in open economy. Similarly to us, they assume log utility to get analytical solutions. Studies showing that demand shocks are an important source of international spillovers include Stockman and Tesar (1995), Pavlova and Rigobon (2007), Wen (2007) and Corsetti et al. (2014).

[5]Other papers which study optimal policy using the same instrument are e.g. Devereux and Engel. (2003) and Liu and Shi (2010).

[6]Therefore, consumption depends on the state of nature but not on the history of the economy or the particular date.

[7]The welfare criterion that we use is weaker than *ex-ante* optimality, which also requires that agents are able to insure against the risk they face when young. See Muench (1977) and Chattopadhyay and Gottardi (1999) for a discussion.

equilibrium is suboptimal, then this implies that exchange rate fluctuations as driven by the fundamentals are inefficient.

Secondly, we show that monetary transfers are not neutral as they affect the equilibrium allocation. As the competitive equilibrium is not CPO, this means that there is scope for monetary policy to implement a better allocation. The main trade-off which will be at the heart of optimal policy both in the non-cooperative and in the cooperative case can be described as follows. On the one hand, an increase in the money supply means a higher wealth for the domestic agents, which incentivize them to work less and to enjoy more leisure. On the other hand, lower working hours mean a lower output, which implies lower consumption not only for domestic agents but also for foreign ones.

Under non-cooperation, each central bank chooses the domestic monetary transfers which maximize a weighted sum of the utility of the domestic agents taking as given the policy of the other central bank. Under the optimal policy, the incentive is to devaluate the domestic currency by setting a positive monetary transfer in all states of nature. In doing so, each central bank imposes a negative spillover to the other country whose consumption of the foreign good is reduced. This attempt is self-defeating as the countries end up in a suboptimal (Nash) equilibrium.

Under cooperation, the welfare function is the same one that we adopt for the planner's problem. The main difference is that the cooperating central banks cannot directly choose the consumption allocation and the labor supply but need to operate through their monetary policy instruments. We show that the two negative spillovers occurring under non-cooperation are internalized and output in both countries is higher. We also show that the two central banks, when cooperating, are able to implement a first-best allocation through their monetary policy instruments. However, this requires that the country which faces the highest global demand for the domestic good is given a higher weight in the social welfare function. The reason is that the agents born in such country need to put a higher labor effort, which decreases their utility. To compensate for that, those agents must be assigned a higher weight. This paper then suggests a novel reason behind the fact that countries find international monetary cooperation difficult[8]. Since the two countries must be treated differently if a first-best allocation is to be implemented, then it is likely that the country weighted less would not agree to cooperate.

Next, we compare the volatility of the nominal exchange rate under three different

[8]See Ostry and Ghosh (2016) for other reasons behind the lack of international cooperation.

scenarios: (1) a situation where central banks are inactive and the exchange rate only moves with the demand shocks; (2) non-cooperation; (3) cooperation. We show that the volatility can be clearly ranked: it is the highest under non-cooperation, while it is the lowest under cooperation. In a non-cooperative environment, it is interesting to note that central banks make exchange rate volatility even worse as compared to a scenario where they are inactive. This result can be explained as follows. In the state of nature where there is a high demand for the domestic good, the domestic currency appreciates in equilibrium as all agents in the world economy require more domestic currency. Under non-cooperation, both central banks increase money supply. However, the domestic central bank pursues a relatively less expansionary policy (compared to the other country) to incentivize labour effort knowing that domestic agents have a high demand for the domestic good. As a result, the currency appreciates even further. Cooperation is then able to achieve two things. Firstly, it eliminates the excess exchange rate volatility created under non-cooperation. Secondly, it curbs the excess volatility due to the incompleteness of the markets. Cooperation implies that the two central banks agree to carry out a relatively more expansionary monetary policy in the country whose currency tends to appreciates because of a relatively high demand for the domestic good (hence, for the domestic currency). In other words, the two central banks would implement policies which "lean against the wind". Another interesting finding is that while cooperation takes care of the excess volatility, it does not implement a fixed exchange rate or a monetary union. This means that some degree of exchange rate volatility can be harmless.

We then look at the welfare consequences of cooperation. In particular, we ask whether moving from the Nash equilibrium to the CPO allocation through cooperation involves welfare improvements for all agents in the world economy. This is not obvious since while cooperation always involves an increase in output, it also implies lower leisure. We prove that all agents would gain in moving from a non-cooperative environment to the CPO allocation for an open set of parameters. However, if countries are sufficiently heterogeneous, we demonstrate that the first-best allocation might not be Pareto improving. In the transition from the Nash equilibrium to the Pareto frontier, one of the two countries could be worse off. However, there exist other (second-best) allocations which can be reached through cooperation and are Pareto improving. This still requires that countries are weighted differently in the social welfare function. In these cases, we show that a utilitarian welfare function which treats agents identically does not deliver an allocation that improves welfare for all agents, making international monetary cooperation basically impossible.

Finally, we investigate whether cooperation can bring relevant welfare gains. For our benchmark parametrization, the welfare gains are equal to 0.6% in terms of equivalent consumption. Although this number is not particularly large, it is far from negligible. This is especially considering that, in a New Keynesian setting, the gains from cooperation are exactly zero when the utility function is logarithmic as in this paper[9]. Hence, our framework suggests that international monetary cooperation among central banks is worthwhile. The excess exchange rate volatility due to the incompleteness of the markets and central banks' incentives in a non-cooperative environment is costly, hence it is in central banks' best interest to cooperate in order to reduce it.

Related literature - This paper speaks to several strands of literature. Firstly, it relates to other general equilibrium models of nominal exchange rate (in)determinacy with flexible prices. To the best of our knowledge, this is the first paper showing that fundamentals-related exchange rate fluctuations are inefficient in a flexible price environment. For instance, exchange rate movements in Lucas (1982) are efficient since markets are complete and the competitive equilibrium of the economy is Pareto optimal. In an OLG model where two currencies can be used to buy a single consumption good, Manuelli and Peck (1990) demonstrated that, for any equilibrium allocation, one can construct many paths of the nominal exchange rate. Hence, the exchange rate volatility arising in their model is irrelevant from a welfare point of view. In two-period models with incomplete markets where assets (in zero net supply) are nominal and denominated in multiple currencies, there is real indeterminacy of the equilibrium allocation so that for each equilibrium we have a different value of the nominal exchange rate (Polemarchakis, 1988). Introducing outside money into the model, Pietra and Salto (2011) have shown that there is no way to Pareto rank the different equilibria and to make a clear-cut statement on exchange rate volatility.

Secondly, this paper is connected to the literature on the gains from cooperation. Our paper has in common with Cooley and Quadrini (2003), Celentani, Conde-Ruiz and Desmet (2006) and Liu and Shi (2010) the investigation of the gains from cooperation in flexible price models with perfect competition. Differently from this paper, countries do not attempt to manipulate the nominal exchange rate *per se* and the welfare issues

[9]See e.g. Corsetti, Dedola and Leduc, 2010.

surrounding the volatility of the exchange rate are not explored[10]. This issue has been extensively studied in economies with sticky prices and imperfect competition. Some key contributions include Obstfeld and Rogoff (2002), Benigno and Benigno (2003, 2006), Corsetti and Pesenti (2001, 2005) just to name a few. The gains from cooperation in New Keynesian models are typically negligible, and are exactly zero when the utility function is logarithmic. This led Obstfeld and Rogoff (2002) to argue that international spillovers from monetary policy are a second-order problem as compared to internal objectives such as price and output stabilization[11].

This paper is also related to recent papers which adopt an OLG framework to analyse "currency wars" at the zero lower bound (Eggertsson et al., 2016; Caballero, Fahri and Gourinchas, 2016). As in this paper, issuances of public debt and helicopter drops are identical policies when the nominal interest rate is zero since the two assets are perfect substitutes. In our model, such policies have a "beggar-thy-neighbour" effect: an increase in domestic money supply decreases foreign consumption by negatively affecting the labour supply of the domestic agent through a wealth effect. On the other hand, in these papers they generate a positive spillover by increasing output in all countries due to the nominal rigidities[12].

The paper is structured as follows. In section 2, we present the model and compute the stationary equilibrium. In section 3, we analyse the planner's problem to identify the set of conditionally Pareto optimal allocations. In section 4, we look at optimal policy under non-cooperation and cooperation. In section 5, we compare the volatility of the nominal exchange rate under different policy scenarios. In section 6, we investigate the welfare consequences of cooperation. Section 7 concludes. All the proofs are in the Appendix.

[10] In Cooley and Quadrini (2003) and Celentani et al. (2006), countries have an incentive to manipulate the terms of trade. In Liu and Shi (2010)' s model with search frictions, countries attempt to exploit deviations from the law of one price.

[11] This point has not been challenged by subsequent literature. See e.g. the literature review of Corsetti et al. (2010). However, Rabitsch (2012) demonstrated in a model with imperfect risk sharing that the gains are of an order of magnitude larger than under complete markets. Differently from her paper, our welfare gains do not rely on the existence of nominal rigidities and can be thought of as long-run gains from cooperation.

[12] 'In Eggertsson et al. (2016) and Caballero et al. (2016), "currency wars" arise instead due to exchange rate policies: since in their framework the nominal exchange rate is indeterminate à la Kareken and Wallace (1981), it can be manipulated.

2 The model

Time is discrete. At each date, a state of nature s realizes. The number of states S is finite. s^t denotes an event and $s^{t+1}|s^t$ is an immediate successor.

There are two countries. In each period, an agent with a two-period lifetime is born in each country. For simplicity, we assume that agents work in the first period and consume in the second. Hence, we abstract from saving decisions. Competitive firms produce a country-specific good, but agents gain utility from the consumption of both goods produced in the world economy as in Lucas (1982). We will use the superscript ℓ to indicate the good or currency of country ℓ, while the subscript h will refer to agents.

Two currencies are available in the world economy. $M^\ell(s^t)$ are the units of currency ℓ in event s^t. We set currency 1 as our numéraire. $e(s^t)$ is the nominal exchange rate or the relative price of currency 2 with respect to currency 1. If $e(s^t)$ rises, we say that the currency of country 2 appreciates with respect to currency 1. Central banks distribute lump-sum (nominal) monetary transfers. $T_h^\ell(s^t)$ denotes the transfers that the old agent born in country h receives from the government of country ℓ in event s^t. For the sake of realism, agents only receive transfers from the domestic government: $T_h^\ell(s^t) = 0$ for $h \neq \ell$. We will study the stationary equilibrium of the model, i.e. an equilibrium allocation where consumption varies across states of nature but does not depend on the history of the economy.

2.1 Firms

Firms produce a country-specific good according to the production function: $y^\ell(s^t) = Z^\ell(s)L^\ell(s^t)$. Labour productivity $Z^\ell(s)$ depends on the state realized and follows a first-order Markov chain. $f(s'|s)$ is the probability that s' realizes conditional on state s (transition probability).

Firms' maximisation problem is then the following:

$$\max_{L^\ell(s^t)} p^\ell(s^t)y^\ell(s^t) - \omega^\ell(s^t)L^\ell(s^t)$$

subject to:

$$y^\ell(s^t) = Z^\ell(s)L^\ell(s^t)$$

where $\omega^\ell(s^t)$ is the nominal wage and $p^\ell(s^t)$ is the price of good ℓ in the event s^t, which are both denominated in units of the domestic currency ℓ. As there is perfect competition, workers are paid their marginal product:

$$\tilde{\omega}^\ell(s^t) \equiv \frac{\omega^\ell(s^t)}{p^\ell(s^t)} = Z^\ell(s)$$

where $\tilde{\omega}^\ell(s^t)$ is the real wage.

We will assume that labour markets are closed, so that firms can only hire domestic workers.

2.2 Central banks

Money supply evolves as follows:

$$M^\ell(s^{t+1}|s^t) = M^\ell(s^t) + T^\ell(s^{t+1}|s^t)$$

In each period, each central bank commits to giving a monetary transfer $T^\ell(s^{t+1}|s^t)$ to the generation born in the current period (and in the domestic economy) when they become old. We assume that the transfer is proportional to the total stock of money supply and such proportion depends on the current state of nature:

$$T^\ell(s^{t+1}|s^t) = \mu^\ell(s) M^\ell(s^t)$$

where $\mu^\ell(s)$ is the monetary policy instrument. Therefore, the law of motion of money supply is:

$$M^\ell(s^{t+1}|s^t) = (1+\mu^\ell(s))M^\ell(s^t) \qquad 1 > \mu^\ell(s) > -1 \tag{1}$$

The monetary transfers can vary across states of nature. However, the transfer is perfectly anticipated from the perspective of the agent that receives it when old since it depends on the state realized when young. The lack of uncertainty about the monetary transfer ensures that the model is fully analytically tractable[13].

The initial stocks of money will be denoted as $\bar{M}^\ell$.

2.3 Households

The utility function of an agent born in country h in state s is:

$$\max_{l_h(s),\mathbf{c}_h(s'|s),\mathbf{m}_h(s)} U_h(s) \equiv -\frac{l_h(s)^{1+\eta}}{1+\eta} + \sum_{s'} f(s'|s)[a_h^1(s)\log c_h^1(s'|s) + a_h^2(s)\log c_h^2(s'|s)] \tag{2}$$

[13] In Lucas (1982), agents face random monetary transfers in the future but it is assumed that there exists a set of claims to all future monetary transfers, so that agents can fully insure against the monetary policy shocks and markets are complete.

where $l_h(s)$ is the labor supply, $\mathbf{c}_h(s'|s) := (c_h^1(s'|s), c_h^2(s'|s))$ is a consumption vector, $\mathbf{m}_h(s) := (m_h^1(s), m_h^2(s))$ is a portfolio of currencies and $a_h^\ell(s)$ is the weight assigned to good ℓ.

The timing works as follows. Firstly, agents earn a wage by supplying their labour (to domestic firms) and spend it buying a portfolio of currencies to fund next period's consumption. Next, the state of nature realizes. After learning about the state of the economy, each agent uses currency 1 (2) to buy good 1 (2).

More formally, agents are subject to the following constraints:

$$
\begin{aligned}
m_h^1(s^t) + e(s^t) m_h^2(s^t) &= w_h(s^t) \\
p^\ell(s^{t+1}|s^t) c_h^\ell(s'|s) &= m_h^\ell(s^t) + T_h^\ell(s^{t+1}|s^t) \qquad \forall\ s^{t+1}, \ell
\end{aligned}
$$

where $w_h(s^t)$ is agent h's nominal wealth. The wealth of agent 1 and 2 is respectively equal to their labour income: $w_1(s^t) \equiv l_1(s)\omega_1(s^t)$ and $w_2(s^t) \equiv e(s^t) l_2(s)\omega_2(s^t)$. In the budget constraint of the young, nominal variables are converted in units of the numéraire (currency 1). $p^\ell(s^{t+1}|s^t)$ is the price of good ℓ tomorrow, conditionally on today's event being s^t. As we previously mentioned, agents only receive a nominal transfer from the domestic central bank: $T_h^\ell(s^{t+1}|s^t) = 0$ if $\ell \neq h$. Note that our timing imply that the old face two budget constraints in each state of nature.

For convenience, we rewrite the budget constraints as follows. The relative price of good ℓ in terms of good 1 (expressed in units of the same currency), otherwise known as the terms of trade of the economy, can be defined as: $\varepsilon(s^t) \equiv \frac{p^2(s^t)e(s^t)}{p^1(s^t)}$. Therefore, we divide the budget constraint of the young by $p^1(s^t)$ and the budget constraint of the old by $p^\ell(s^{t+1}|s^t)$ and use $\varepsilon(s^t)$ to rewrite the constraints as:

$$
\tilde{m}_h^1(s^t) + \varepsilon(s^t)\tilde{m}_h^2(s^t) = \tilde{w}_h(s^t) \tag{3}
$$

$$
c_h^\ell(s'|s) = \frac{\tilde{m}_h^\ell(s^t)}{1 + \pi^\ell(s^{t+1}|s^t)} + \tilde{T}_h^\ell(s^{t+1}|s^t) \qquad \forall\ s^{t+1}, \ell \tag{4}
$$

where $\tilde{m}_h^\ell(s^t) \equiv \frac{m_h^\ell(s^t)}{p^\ell(s^t)}$, $\tilde{T}_h^\ell(s^{t+1}|s^t) \equiv \frac{T_h^\ell(s^{t+1}|s^t)}{p^\ell(s^{t+1}|s^t)}$ and $\pi^\ell(s^{t+1}|s^t)$ is the inflation rate in terms of good ℓ. The real wealth of agent 1 and 2 are respectively $\tilde{w}_1(s^t) \equiv l_1(s)\tilde{\omega}_1(s^t)$ and $\tilde{w}_2(s^t) \equiv \varepsilon(s^t) l_2(s)\tilde{\omega}_2(s^t)$. The solution to the above maximisation problem can be found in the Appendix.

2.4 Stationary equilibrium

Definition 1 *A stationary equilibrium is a system of prices $\varepsilon(s^t) \in \mathbb{R}_{++}$, $p(s^0) \in \mathbb{R}^2_{++}$ and $\pi^\ell(s^t) \in \mathbb{R}^2_{++}$ for every s^t, labour supply $l_h(s)$ for every h and s, consumption allocations $\mathbf{c}_h(s'|s) \in \mathbb{R}^2_{++}$ for every h and any pair of s', s and portfolio allocations $\tilde{\mathbf{m}}_h(s^t) \in \mathbb{R}^2_{++}$ for every s^t such that:*

(i) Agent h maximizes his utility function (2) subject to the budget constraints (3) and (4) in every s^t;

(ii) Firms maximise their profits for every s^t and ℓ;

(iii) $\sum_h c_h^\ell(s|s') = y^\ell(s) \qquad \forall\ s, s'$ *and* $\forall\ \ell$

(iv) $\sum_h \tilde{m}_h^\ell(s^t) = \tilde{M}^\ell(s^t) \qquad \forall\ s^t, \ell$

(v) $L^1(s^t) = l_1(s) \qquad L^2(s^t) = l_2(s)$

As labour markets are closed, labour demand is already pinned down by the other conditions. Hence, conditions (v) are already satisfied. Moreover, that also implies that the real wage of each agent is equal to domestic productivity: $\tilde{\omega}_1(s^t) = Z^1(s)$ and $\tilde{\omega}_2(s^t) = Z^2(s)$.

In the next Proposition, we show how we can considerably simplify and reduce the equilibrium system to get a closed form solution for the stationary equilibrium of the economy.

Proposition 1 *The stationary equilibrium of the economy is fully characterized by the following equations:*

$$\begin{array}{ll} l_1(s) = \frac{1}{(1+\mu^1(s))^{\frac{1}{1+\eta}}} & l_2(s) = \frac{1}{(1+\mu^2(s))^{\frac{1}{1+\eta}}} \\ c_1^1(s'|s) = \frac{a_1^H(s)Z^1(s')}{(1+\mu^1(s'))^{\frac{1}{1+\eta}}} & c_1^2(s'|s) = \frac{a_2^F(s)Z^2(s')}{(1+\mu^2(s'))^{\frac{1}{1+\eta}}} \\ c_2^1(s'|s) = \frac{a_1^F(s)Z^1(s')}{(1+\mu^1(s'))^{\frac{1}{1+\eta}}} & c_2^2(s'|s) = \frac{a_2^H(s)Z^2(s')}{(1+\mu^2(s'))^{\frac{1}{1+\eta}}} \end{array} \tag{5}$$

Proposition 1 shows that monetary policy is not neutral in our model, as each central bank can affect the labor supply decision of the domestic agents by changing $\mu^\ell(s)$. If the young know that they will receive a positive transfer from the domestic central bank when old, then they have an incentive to work less because of a positive wealth effect. However, output in the two countries would decrease, hence a higher monetary transfer would have a detrimental effects in terms of consumption for both countries. This is the basic trade-off that optimal policy will face, both in non-cooperative and cooperative environments.

Let us now derive the equation which shows the behaviour of the nominal exchange rate in equilibrium. In the Appendix, we derive the equilibrium terms of trade of the economy, which depend on the productivity and the demand shocks, as well as on the monetary policy transfers:

$$\varepsilon(s) = \frac{a_1^2(s)}{a_2^1(s)} \frac{Z^1(s)}{Z^2(s)} \left(\frac{1+\mu^1(s)}{1+\mu^2(s)} \right)^{\frac{\eta}{1+\eta}}$$

We also show that the real money balances of each currency are equal to the domestically produced output. Hence, the two nominal prices at each event can be calculated as follows:

$$p^\ell(s^t) = \frac{M^\ell(s^t)}{y^\ell(s)} = \frac{M^\ell(s^t)}{Z^\ell(s)L^\ell(s)} = \frac{M^\ell(s^t)(1+\mu^\ell(s))^{\frac{1}{1+\eta}}}{Z^\ell(s)}$$

using the solution for labour supplies above. The inflation rates between any two periods are therefore:

$$\pi^\ell(s'|s) = \frac{p^\ell(s^{t+1}|s^t)}{p^\ell(s^t)} - 1 = \frac{Z^\ell(s)}{Z^\ell(s')}(1+\mu^\ell(s')) - 1 \tag{6}$$

Price growth depends on productivity growth as well as on the current monetary transfer. Notice that each central bank will indirectly set the rate of inflation by choosing $\mu^\ell(s')$ optimally.

Given the above, the nominal exchange rate obeys the following equation:

$$e(s^t) \equiv \frac{\varepsilon(s)p^1(s^t)}{p^2(s^t)} = \frac{M^1(s^t)}{M^2(s^t)} \frac{a_1^2(s)}{a_2^1(s)} \frac{1+\mu^1(s)}{1+\mu^2(s)} \tag{7}$$

The behaviour of the nominal exchange rate depends on three sets of variables: (1) the relative money supplies, which carry information about the history of the economy; (2) the relative demand for the foreign good in the current state of nature; (3) the relative monetary transfers in the current state.

In particular, currency 1 depreciates (e rises) whenever the current demand for the foreign good is relatively higher with respect to the past and the domestic central bank's monetary policy is more expansionary. As the demand for the foreign good rises with respect to the past, agents will demand more foreign currency leading to the domestic currency's depreciation. An expansionary monetary policy means that, as domestic agents expect to receive a transfer in the future, they have an incentive to work less and output falls. This implies that the domestic good becomes relatively more expensive. But then, agents have an incentive to demand less currency 1 as they expect higher inflation in good 1. Therefore, currency 1 depreciates in equilibrium.

We conclude this section by discussing why a stationary equilibrium exists in this framework. This is not obvious given the result of Spear (1985), who showed that a stationary equilibrium does not generically exist in pure exchange stochastic OLG economies with multiple goods. In an OLG model, S^2 equations need to clear for each good as the aggregate consumption $c^\ell(s|s')$ depends not just on the current state s but also on the state in which agents are born s' (see Definition 1). In Spear (1985)'s closed economy framework, the number of equations that need to clear are then $(L-1)S^2$ in the goods' markets (once one applies Walras Law) and S equations in the money market, while the number of unknowns is LS, i.e. the prices of the two goods expressed in currency units. Spear's non-existence result is then related to the fact that there are too many equations with respect to the number of unknowns. Notice that this issue does not arise in a model with one commodity ($L = 1$).

In our two-currency economy, the number of equations which need to clear is even higher as the number of equations in the money markets is $2S$ instead of S. The number of unknowns is also higher as, besides $2S$ nominal prices, we need to solve for S relative prices i.e. to solve for the terms of trade in each state of nature (nominal exchange rates are pinned down as a result). This still leaves us with too many equations with respect to the number of unknowns. However, we show in Step 3 of Proposition 1 that only S equations in the goods' markets are actually independent. Since each currency can only buy the local good, the real money balances of each currency are equal to the domestic output, hence they only depend on the current state:

$$y^\ell(s) = \tilde{M}^\ell(s)$$

Using the budget constraints, we can then argue that the aggregate consumption of each good does not depend on the past:

$$c^\ell(s|s') = \tilde{M}^\ell(s) \qquad \Rightarrow \qquad c^\ell(s|s') = c^\ell(s)$$

This means that we can get rid of $S^2 - S$ equations in the goods' markets. As a consequence, we are left with an equilibrium system of $S + 2S$ independent equations and unknowns. This implies that a stationary equilibrium exists.

3 (Conditionally) Pareto optimal allocations

Before analysing optimal policy in our framework, we characterise the set of Pareto optimal allocation. In order to do so, we use the concept of Conditional Pareto Optimality (CPO), first proposed by Muench (1977). Under this welfare criterion, in a stationary equilibrium agents are distinguished not only by type but also by the event occurring at their birth. CPO allocations have the property that, conditionally on being born in a particular state of nature, agents can insure against all risks they face *after* they are born. Hence, markets are said to be sequentially complete. Given the sequential nature of trading, the notion that agents can also insure against the risk they face when they are born (*ex-ante* optimality) is highly unrealistic. CPO optimality, although weaker than *ex-ante* optimality, seems a more suitable benchmark in an OLG framework[14].

Hence, we set up a social welfare function where agents are distinguished by their country of birth and the state of nature they face when they are born. In particular, we assume that the planner can directly choose the consumption allocation and the labour supply of all agents so to maximise a weighted sum of the utility of all agents, subject to the feasibility constraints and the production functions of the two countries:

$$\max_{\mathbf{c}_1(s|s'),\mathbf{c}_2(s|s'),l_1(s),l_2(s)} W^P = \sum_s \gamma_1^P(s)U_1(s) + \sum_s \gamma_2^P(s)U_2(s) \tag{8}$$

where $\sum_s \gamma_1^P(s) + \sum_s \gamma_2^P(s) = 1$. Agent 1 and agent 2's consumption allocations are respectively $\mathbf{c}_1(s|s') := (c_1^1(s|s'), c_1^2(s|s'))$ and $\mathbf{c}_2(s|s') := (c_2^1(s|s'), c_2^2(s|s'))$. This is subject to:

$$c_1^1(s|s') + c_2^1(s|s') = y^1(s) \quad \forall\ s, s' \tag{9}$$

$$c_1^2(s|s') + c_2^2(s|s') = y^2(s) \quad \forall\ s, s' \tag{10}$$

$$y^1(s) = Z^1(s)l_1(s) \quad \forall\ s \tag{11}$$

$$y^2(s) = Z^2(s)l_2(s) \quad \forall\ s \tag{12}$$

From now onwards, let us define the weight attached to the domestic good as the degree of home bias of a country, which can be interpreted as the degree of openness of the economy: $a_1^1(s) \equiv a_1^H(s)$ and $a_2^2(s) \equiv a_2^H(s)$, where $a_h^F(s) \equiv 1 - a_h^H(s)$ for $h = 1, 2$.

[14]See Chattopadhyay and Gottardi (199) for a discussion.

In the Appendix, we show how to find the set of CPO allocations:

$$l_1^P(s) = \left[\frac{\sum_{s'} f(s|s')(\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s'))}{\gamma_1^P(s)}\right]^{\frac{1}{1+\eta}} \tag{13}$$

$$c_1^{1P}(s|s') = \frac{\gamma_1^P(s')a_1^H(s')}{\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s')} l_1^P(s)Z^1(s) \tag{14}$$

$$c_2^{1P}(s|s') = \frac{\gamma_2^P(s')a_2^F(s')}{\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s')} l_1^P(s)Z^1(s) \tag{15}$$

$$l_2^P(s) = \left[\frac{\sum_{s'} f(s|s')(\gamma_2^P(s')a_2^H(s') + \gamma_1^P(s')a_1^F(s'))}{\gamma_2^P(s)}\right]^{\frac{1}{1+\eta}} \tag{16}$$

$$c_2^{2P}(s|s') = \frac{\gamma_2^P(s')a_2^H(s')}{\gamma_2^P(s')a_2^H(s') + \gamma_1^P(s')a_1^F(s')} l_2^P(s)Z^2(s) \tag{17}$$

$$c_1^{2P}(s|s') = \frac{\gamma_1^P(s')a_1^F(s')}{\gamma_2^P(s')a_2^H(s') + \gamma_1^P(s')a_1^F(s')} l_2^P(s)Z^2(s) \tag{18}$$

The higher is the weight assigned to an agent born in a given state s, the lower will be the amount of labour that the planner will require such agent to supply since labour effort decreases utility. On the other hand, labour supply in a given country (and therefore, output) increases with the weights of all the agents that might consume the good in state s and their relative preference for the good. In allocating the output of each good across agents, the planner takes into account the weights given to the two agents as well as their relative preference for the good.

3.1 Sequentially incomplete markets

The following Proposition shows that the competitive equilibrium of the economy is CPO only in very special cases. In other words, markets are generically sequentially incomplete[15].

Firstly, let us introduce the following definition:

Definition 2 $\frac{a_2^F(s)}{a_1^F(s)} \equiv A(s)$.

Definition 3 *When $A(s) \equiv A$, we say that demand shocks are perfectly symmetric.*

Definition 3 implies that $\frac{a_2^F(s)}{a_2^F(s')} = \frac{a_1^F(s)}{a_1^F(s')}$ for every s, s'. This means that if e.g. there is an increase in demand for the foreign good in country 2, an identical increase in demand for the foreign good will occur in country 1.

[15] For competitive equilibrium, we refer to a situation where monetary authorities are inactive after the first period: $\mu^1(s) = \mu^2(s) = 0$.

Proposition 2 *Suppose that central banks are inactive ($\mu^1(s) = \mu^2(s) = 0$). The competitive equilibrium of the economy is CPO if and only if demand shocks are perfectly symmetric.*

The above Proposition shows that markets are effectively sequentially complete (i.e. the competitive equilibrium is a CPO allocation) only if demand shocks are perfectly symmetric across countries, i.e. the change in demand for the foreign good across states is the same across countries. The condition for conditional Pareto optimality identified above includes the following cases: (1) preferences do not vary over time within country: $a_h^F(s) = a_h^F$ for every h; (2) preferences are identical across countries: $a_1^F(s) = a_2^F(s)$ for every s.

It is interesting to observe that, if the fundamentals of the economy are such that markets are effectively sequentially complete, the exchange rate is constant across states of nature. In fact, equation (7) would become:

$$e(s^t) = \bar{e} = \frac{\bar{M}^1}{\bar{M}^2} A$$

More generally, the competitive equilibrium is not CPO: typically, the economy will exhibit exchange rate volatility and such volatility is suboptimal.

This result has important implications as it shows that, generically, there is scope for monetary policy to improve on the competitive equilibrium. Whether monetary policy is able to restore conditional Pareto optimality and under which conditions, as well as the implications for the behaviour of the nominal exchange rate, is the subject of investigation of the following sections.

4 Optimal monetary policy

Firstly, we analyse the case where each central bank only cares about the welfare of domestic residents. Therefore, we set up a non-cooperative game between the two central banks. We assume that each central bank forms an expectation about the monetary policy adopted by the other central bank and, taking the policy of the other central bank as given, chooses the vector of monetary transfers that maximises its welfare function, which is a weighted sum of the utility function of the domestic agents born in different states.

The welfare functions of the two central banks can be written as follows:

$$W_1 = \sum_s \gamma_1^N(s) U_1(s) \tag{19}$$

$$W_2 = \sum_s \gamma_2^N(s) U_2(s) \tag{20}$$

where $\sum_s \gamma_h^N(s) = 1$. Let us recall that the labour supply decisions and the consumption allocations in the decentralized economy for agents born in country 1 and 2 are respectively[16]:

$$\begin{aligned} l_1(s) &= \frac{1}{(1+\mu^1(s))^{\frac{1}{1+\eta}}} \\ c_1^1(s'|s) &= \frac{a_1^H(s) Z^1(s')}{(1+\mu^1(s'))^{\frac{1}{1+\eta}}} \\ c_1^2(s'|s) &= \frac{a_2^F(s) Z^2(s')}{(1+\mu^2(s'))^{\frac{1}{1+\eta}}} \end{aligned} \tag{21}$$

$$\begin{aligned} l_2(s) &= \frac{1}{(1+\mu^2(s))^{\frac{1}{1+\eta}}} \\ c_2^1(s'|s) &= \frac{a_1^F(s) Z^1(s')}{(1+\mu^1(s))^{\frac{1}{1+\eta}}} \\ c_2^2(s'|s) &= \frac{a_2^H(s) Z^2(s')}{(1+\mu^2(s))^{\frac{1}{1+\eta}}} \end{aligned} \tag{22}$$

Proposition 3 *The unique Nash equilibrium of the non-cooperative monetary policy game between the central banks is:*

$$\mu^{1N}(s) = \frac{\gamma_1^N(s)}{\sum_{s'} f(s|s') \gamma_1^N(s') a_1^H(s')} - 1 \qquad \forall\ s \tag{23}$$

$$\mu^{2N}(s) = \frac{\gamma_2^N(s)}{\sum_{s'} f(s|s') \gamma_2^N(s') a_2^H(s')} - 1 \qquad \forall\ s \tag{24}$$

The monetary transfer in a given state of nature depends on three variables: (1) the home bias of all domestic agents; (2) the conditional probabilities that such state of nature occurs; (3) the distribution of weights assigned to the domestic agents. Firstly, the higher is the degree of home bias of domestic agents the lower is the monetary transfer. If domestic agents attach a high weight to the domestic good, the central bank has an incentive to boost domestic output and therefore it decreases the monetary transfers so

[16] See Proposition 1.

that domestic agents supply more labour. Secondly, if a particular state of nature is more likely to occur conditionally on the other states, then agents have a high probability to end up in that particular state when old and therefore domestic output must be increased by decreasing the monetary transfer. Finally, if the central bank assigns a higher weight to the agent born in a particular state as compared to the others, then the monetary transfer would rise so as to increase the leisure (and therefore utility) of that agent.

Notice that each central bank has the incentive to set a positive rate of money growth, hence to generate inflation (equation (6)) and an exchange rate depreciation (equation (7)) as compared to a situation where central banks are inactive. It can also be observed that central banks' actions impose a negative spillover to the other country, as the consumption of the foreign good would fall (equation (5)).

Let us now suppose that the two central banks cooperate delegating their decisions to a world central bank that chooses the monetary transfers in the two countries $\mu^{\ell}(s)$ to maximise the following global welfare function:

$$W = \sum_{s} \gamma_1^C(s) U_1(s) + \sum_{s} \gamma_2^C(s) U_2(s) \tag{25}$$

where $\sum_s \gamma_1^C(s) + \sum_s \gamma_2^C(s) = 1$. This is subject to the equilibrium equations (21) and (22)[17].

Proposition 4 *Under cooperation, the optimal monetary transfers are:*

$$\mu^{1C}(s) = \frac{\gamma_1^C(s)}{\sum_{s'} f(s|s')[\gamma_1^C(s') a_1^H(s') + \gamma_2^C(s') a_2^F(s')]} - 1 \qquad \forall\ s \tag{26}$$

$$\mu^{2C}(s) = \frac{\gamma_2^C(s)}{\sum_{s'} f(s|s')[\gamma_2^C(s') a_2^H(s') + \gamma_1^C(s') a_1^F(s')]} - 1 \qquad \forall\ s \tag{27}$$

The crucial difference with the non-cooperative game is that the cooperating central banks take into account the demand of each country for the foreign good. Hence, they internalize the negative effect that an expansionary monetary policy has on the consumption level (and therefore utility) of foreign agents. As a result, the optimal monetary transfers are always lower under cooperation. This implies that aggregate output in the two countries would always increase if we moved from a Nash equilibrium to a cooperative

[17]Notice that while the social welfare function is the same as the planner's in section 3, central banks cannot directly choose the consumption allocation and labour supply but have to operate with their monetary policy instruments, subject to the equilibrium restrictions.

environment. However, higher output means that agents are required to work more and enjoy less leisure. In a later section, we will then ask whether cooperation actually implies welfare gains for every agent in the economy.

Let us now study how the allocations which can be implemented by central banks through optimal policy relate to CPO allocations.

Firstly, we check our intuition that non-cooperative policies lead to a suboptimal equilibrium proving the following result:

Proposition 5 *The Nash equilibrium is conditionally Pareto suboptimal.*

Secondly, we compare the allocations which can be achieved through cooperation with CPO allocations.

Proposition 6 *Cooperative policies can implement the first-best allocation for weights in the social welfare function (25) equal to:* $\gamma_1^C(s) = \gamma_1^P(s)$, $\gamma_2^C(s) = \gamma_2^P(s)$ *and* $\frac{\gamma_1^C(s)}{\gamma_2^C(s)} = \frac{a_2^F(s)}{a_1^F(s)}$ *for every* s.

Corollary 1 *The implementation of a first-best allocation requires that the domestic and the foreign agent are weighted differently, unless they are identical.*

Proposition 6 shows that cooperation between the two central banks can effectively "complete the markets", as it is possible to reach a first-best, CPO allocation through the monetary policy instruments. This can be done by choosing an appropriate vector of weights in the welfare function. More generally, the first and the second-best frontiers are not identical since central banks are constrained by their monetary policy instruments. While the set of output allocations which can be implemented through cooperation are identical to those achievable by the planner, the two central banks are generically not able to reach a first-best consumption distribution unless they choose weights in a certain manner[18]. In order to achieve that, they should give a higher weight to the agent with the lowest demand for the foreign good (or the economy which is relatively more closed). Since this implies that the global demand for the domestically produced good is higher, then the agent is required to work relatively more than the other agent to satisfy the demand. Hence, his utility loss (deriving from the fall in leisure) should be compensated by giving him a higher weight in the welfare function. One of the main implications of the model is that the domestic and the foreign agent should not be treated equally if one

[18]See the Proof for more details.

wants to implement the first-best. Only if countries have the same degree of home bias, then a utilitarian welfare function would enable the two central banks to reach the Pareto frontier through their monetary policy instruments.

Our result then points at a novel reason behind the lack of monetary policy cooperation among central banks. In order to cooperate, it goes without saying that central banks should agree on the social welfare function to maximise. Although the prospect of effectively "completing the markets" appealing, it is unlikely that the central bank of that country whose agent must be assigned a lower weight would agree to the monetary policies associated with the implementation of the first-best. In section 6, we will discuss this point further when analyzing the welfare consequences of moving from the Nash equilibrium to a cooperative outcome.

5 Exchange rate volatility and welfare

In this section, our objective is to draw some welfare conclusions on the behaviour of the nominal exchange rate. Equation (7) shows that there are two relevant sources of shocks in the model: demand shocks and monetary policy shocks. In the case of non-cooperation, the monetary transfers are set optimally. However, does central banks' competition increase or decrease the volatility of the exchange rate as compared to a scenario where central banks are inactive and the exchange rate only moves with the demand shocks? Moreover, what would cooperation among central banks imply for the volatility of the exchange rate? The aim of this section is to answer these questions by comparing the volatility of the nominal exchange rate under three different scenarios: (1) non-cooperation; (2) cooperation; (3) only demand shocks (inactive central banks).

Notice that the nominal exchange rate is not a stationary variable: while it depends on the current realization of the demand and the monetary policy transfers, it also depends on the history of the economy as summarized by the current stocks of money supplies. To calculate volatility, it is then convenient to transform the variable and make it stationary. We will follow two approaches in calculating the volatility of the nominal exchange rate. Firstly, we make the following normalisation:

Definition 4 $\tilde{e}(s) \equiv \frac{e(s^t)M^2(s^t)}{M^1(s^t)}$.

We basically ask, given the same history of the economy, whether the exchange rate is more or less volatile as a result of the possible values that monetary transfers can take under different policy scenarios. This comparison works very well at $t = 0$, since the money stocks are identical across regimes. We are aware that calculating exchange rate volatility on the basis of this measure is imperfect: at any other moment in time, the relative money supplies would be different across regimes for the same history of the shocks. However, the advantage of this definition is that volatility can be computed analytically. As a result, it will provide a lot of intuition on how central banks respond to demand shocks both under non-cooperation and cooperation.

After showing a ranking of exchange rate volatility under the three policy environments, we turn to a definition of volatility that takes into account the paths of money supplies. In particular, we calculate volatility as the standard deviation of the first-difference of the logarithm of the nominal exchange rate (see e.g. Tenreyro, 2007). We find numerically that the ranking that we obtain under Definition 4 is still valid under this transformation of the nominal exchange rate. For robustness purposes, we consider both the case of symmetric and asymmetric shocks.

5.1 Asymmetric shocks

Firstly, we analyse the case of asymmetric shocks. To make some progress on the issue of exchange rate volatility, we must make some assumptions on the weights selected by the central banks under optimal policy, as well as specify the parameters of the economy a bit further. Under non-cooperation, we assume that each central bank treats domestic agents equally:

$$\gamma_h^N(s) = \gamma_h^N$$

In the case of cooperation, we consider the case where the two central banks decide to implement the CPO allocation, hence they set weights as specified by Proposition 6.

We then assume that the two central banks decide to assign the same weight to agents born in country 1 but in different states[19]:

$$\gamma_1^{CPO}(s) = \gamma_1^{CPO}$$

To facilitate the calculation of volatility, we consider a scenario where there are only two

[19] Note that weights in country 2 are functions of $\gamma_1^{CPO}(s)$ according to Proposition 6. Since different values for $\gamma_1^{CPO}(s)$ would correspond to a different CPO allocation, we need to make an assumption on the weights assigned to agents born in country 1 to select a particular allocation and compare with the other policy scenarios.

states of nature and the transition probability matrix is specified as follows:

$$\Pi = \begin{pmatrix} f(1|1) & f(2|1) \\ f(1|2) & f(2|2) \end{pmatrix} = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$$

Moreover, demand in the two countries evolves as follows:

$$\begin{aligned} a_1^H(1) &= a_1^H + z_1 \\ a_1^H(2) &= a_1^H - z_1 \\ a_2^H(1) &= a_2^H - z_2 \\ a_2^H(2) &= a_2^H + z_2 \end{aligned}$$

where $z_h > 0$. Therefore, an increase in the demand for the domestic good in one country corresponds to a decrease in the demand for the domestic good in the other.

We then show that the variance of the nominal exchange rate $\tilde{e}$ in the three scenarios can be ranked as follows:

Proposition 7 *When shocks are asymmetric,* $VAR(\tilde{e}^N) > VAR(\tilde{e}^D) > VAR(\tilde{e}^{CPO}) \neq 0$.

$\tilde{e}^N$ denotes the nominal exchange rate under non-cooperation, $\tilde{e}^{CPO}$ refers to the exchange rate in a situation where the two central banks implement the first-best allocation, while $\tilde{e}^D$ is the exchange rate in a scenario where the two central banks are inactive ($\mu^1(s) = \mu^2(s) = 0$) and the exchange rate only moves with the demand shocks.

In order to give some intuition about the result, it is useful to show the realisations of the nominal exchange rate under the non-cooperative and cooperative scenarios:

$$\begin{aligned}
\tilde{e}_N(1) &= \underbrace{\frac{1-a_1^H-z_1}{1-a_2^H+z_2}}_{\text{demand in state 1}} \cdot \underbrace{\frac{a_2^H-z_2(2p-1)}{a_1^H+z_1(2p-1)}}_{\text{non-cooperative policies in state 1}} \\
\tilde{e}_N(2) &= \underbrace{\frac{1-a_1^H+z_1}{1-a_2^H-z_2}}_{\text{demand in state 2}} \cdot \underbrace{\frac{a_2^H+z_2(2p-1)}{a_1^H-z_1(2p-1)}}_{\text{non-cooperative policies in state 2}} \\
\tilde{e}_{CPO}(1) &= \underbrace{\frac{1-a_1^H-z_1}{1-a_2^H+z_2}}_{\text{demand in state 1}} \cdot \underbrace{\frac{p\frac{1-a_1^H-z_1}{1-a_2^H+z_2}+(1-p)\frac{1-a_1^H+z_1}{1-a_2^H-z_2}}{\frac{1-a_1^H-z_1}{1-a_2^H+z_2}}}_{\text{cooperative policy in state 1}} = p\tilde{e}_D(1)+(1-p)\tilde{e}_D(2) \\
\tilde{e}_{CPO}(2) &= \underbrace{\frac{1-a_1^H+z_1}{1-a_2^H-z_2}}_{\text{demand in state 2}} \cdot \underbrace{\frac{p\frac{1-a_1^H+z_1}{1-a_2^H-z_2}+(1-p)\frac{1-a_1^H-z_1}{1-a_2^H+z_2}}{\frac{1-a_1^H+z_1}{1-a_2^H-z_2}}}_{\text{cooperative policy in state 2}} = p\tilde{e}_D(2)+(1-p)\tilde{e}_D(1)
\end{aligned}$$

Proposition 7 shows that competition among central banks create even more volatility than demand shocks alone. The intuition behind this is the following. In state 1, it can observed that currency 1 appreciates (e is low) since there is a growth in demand for good 1. This is caused by an increase both in domestic and foreign demand: not only domestic agents have a higher demand for the domestic good but foreign agents have a lower demand for the domestic good (hence, higher for the foreign good). An increase in the global demand for good 1 means that currency 1 is more desirable, hence it appreciates in equilibrium. In a non-cooperative environment, both central banks would implement an expansionary policy. However, the monetary policy in country 1 is less expansionary as compared to country 2 since agents born in country 1 need to work more to satisfy the increase in the demand (although they only care about domestic demand). Hence, currency 1 appreciates even further. When central banks cooperate, agents born in country 1 carry a higher weight in the social welfare function. As we explained before, it is recognised that their leisure falls because they need to work more to satisfy the increase in the demand for the good they produce. As a consequence, monetary policy becomes relatively more expansionary in country 1 to counteract the effect of the positive demand shock. The opposite occurs instead in state 2.

Under cooperation, central banks "lean against the wind" as they carry out a relatively more expansionary policy in the country whose currency appreciates. Notice also that central banks implement a sort of "exchange rate smoothing", since the nominal exchange rate under cooperation is a weighted average of the exchange rate in the two states of nature (when volatility is only triggered by demand shocks). Unless the two states are equally as likely ($p = \frac{1}{2}$), central banks would not smooth the demand shocks perfectly but they would take into account the degree of persistence in the economy[20].

Finally, we simulate the model 1,000 times and calculate the average standard deviation of the first-difference of the log of the nominal exchange rate. We assume that the shock and the home bias parameter are the same across countries: $z_1 = z_2 = z$ and $a_1^H = a_2^H = a^H$. The shock is a 1% increase in demand for the domestic good, and the home bias is set to $a^H = 0.8$. The results are reported in Table 1. It can be observed that the ranking that we provided in Proposition 7 is still valid once we take into account the different paths of money supply. As expected, volatility decreases with the degree of persistence

[20]Central banks would also smooth the shocks perfectly if the stochastic process was i.i.d. In fact, $p = \frac{1}{2}$ is just an example of an i.i.d. process.

Table 1: Volatility of the nominal exchange rate under different policy scenarios: asymmetric shocks

	$p = 0.5$	$p = 0.6$	$p = 0.7$	$p = 0.8$	$p = 0.9$
Nash equilibrium	0.1131	0.1038	0.0923	0.0778	0.0576
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Demand shocks only	0.1012	0.1013	0.0877	0.0717	0.0505
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Cooperation	0.08	0.0784	0.0733	0.0641	0.0479
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Notes. (a) Exchange rate volatility is defined as the standard deviation of the first-difference of the logarithm of the exchange rate $e(s^t)$. (b) Exchange rate volatility is calculated as the average standard deviation of the variable across 1,000 simulations. (c) Standard errors are reported in parentheses. (d) The initial state is assumed to be 1. The parameter values are chosen as follows: $M^1(s^0) = M^2(s^0) = 1$, $a_1^H = a_2^H = 0.8$ and $z_1 = z_2 = 0.01 \times a^H$.

(p) in the economy.

Table 1 shows that the nominal exchange rate is volatile even in a CPO allocation. Hence, our model suggests that some volatility of the nominal exchange rate can be harmless. However, cooperation brings this volatility to a minimum by taking care of the excess volatility due to the demand shocks as well as the incentives of central banks' in a non-cooperative environment.

5.2 Symmetric shocks

In this section, we consider the case of symmetric shocks.

We keep the same setting as above but demand shocks are now specified as follows:

$$
\begin{aligned}
a_1^H(1) &= a_1^H + z_1 \\
a_1^H(2) &= a_1^H - z_1 \\
a_2^H(1) &= a_2^H + z_2 \\
a_2^H(2) &= a_2^H - z_2
\end{aligned}
$$

In other words, the two countries simultaneously experience either an increase or a decrease of demand for the domestic good.

Proposition 8 *When shocks are symmetric,* $VAR(\tilde{e}^N) > VAR(\tilde{e}^D) > VAR(\tilde{e}^{CPO}) \neq 0$.

Proposition 8 shows that the volatility ranking obtained in the case of asymmetric shocks also applies to the case of symmetric shocks. In state 1, both countries face an increase in demand for the domestic good. Whether the domestic currency will appreciate or not will depend on the relative size of the shock as well as the *ex-ante* levels of home bias. Suppose that the countries are *ex-ante* identical but country 1 faces a relatively larger shock. As a result, currency 1 will appreciate in state 1 and depreciate in state 2. Both central banks would carry out a less expansionary policy in state 1 while a more expansionary policy in state 2, in order to incentivize domestic agents to work more in state 1 and less in state 2. Therefore, what would be the net effect on the nominal exchange rate? As country 1 faces a relatively larger shock, then its policies need to be more aggressive: hence, currency 1 would appreciates (depreciates) even more in state 1 (2) as compared to the case where central banks are inactive. As in the case of asymmetric shocks, cooperation would just smooth the effects of the demand shocks on the nominal exchange rate away. Therefore, for e.g. country 1, cooperation implies a relatively more expansionary policy in state 1 and a less expansionary one in state 2 as compared to country 2.

In Table 2, we compute the volatility of the nominal exchange rate under the three scenarios. As above, we assume that the two countries have an *ex-ante* level of home bias equal to 0.8. However, the size of the shock in the two countries needs to be different for demand shocks to generate any volatility of the nominal exchange rate. Therefore, we assume that country 1 faces an increase (decrease) of 2% in the demand for the domestic good in state 1 (2), while country 2 only of 1%.

Since shocks are symmetric, the nominal exchange rate is less volatile as compared to the case of asymmetric shocks. However, it is still the case that non-cooperation produces the highest volatility, while cooperation the lowest.

6 From the Nash equilibrium to a cooperative outcome

In section 4, we have shown that output is always higher under cooperation. However, higher output does not only imply higher consumption but also lower leisure. In this section, we will then ask two main questions. Firstly, is moving from the Nash equilibrium to a CPO allocation through cooperation Pareto improving? In other words, do all agents gain from moving from the Nash equilibrium to a cooperative allocation? Secondly, is it really worth cooperating from a quantitative point of view?

Table 2: Volatility of the nominal exchange rate under different policy scenarios: symmetric shocks

	$p = 0.5$	$p = 0.6$	$p = 0.7$	$p = 0.8$	$p = 0.9$
Nash equilibrium	0.0568	0.0520	0.0462	0.0390	0.0289
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Demand shocks only	0.0568	0.0508	0.0439	0.0359	0.0254
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Cooperation	0.0401	0.0393	0.0368	0.0321	0.0241
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Notes. (a) Exchange rate volatility is defined as the standard deviation of the first-difference of the logarithm of the exchange rate $e(s^t)$. (b) Exchange rate volatility is calculated as the average standard deviation of the variable across 1,000 simulations. (c) Standard errors are reported in parentheses. (d) The initial state is assumed to be 1. The parameter values are chosen as follows: $M^1(s^0) = M^2(s^0) = 1$, $a_1^H = a_2^H = 0.8$, $z_1 = 0.02 \times a^H$, and $z_2 = 0.01 \times a^H$.

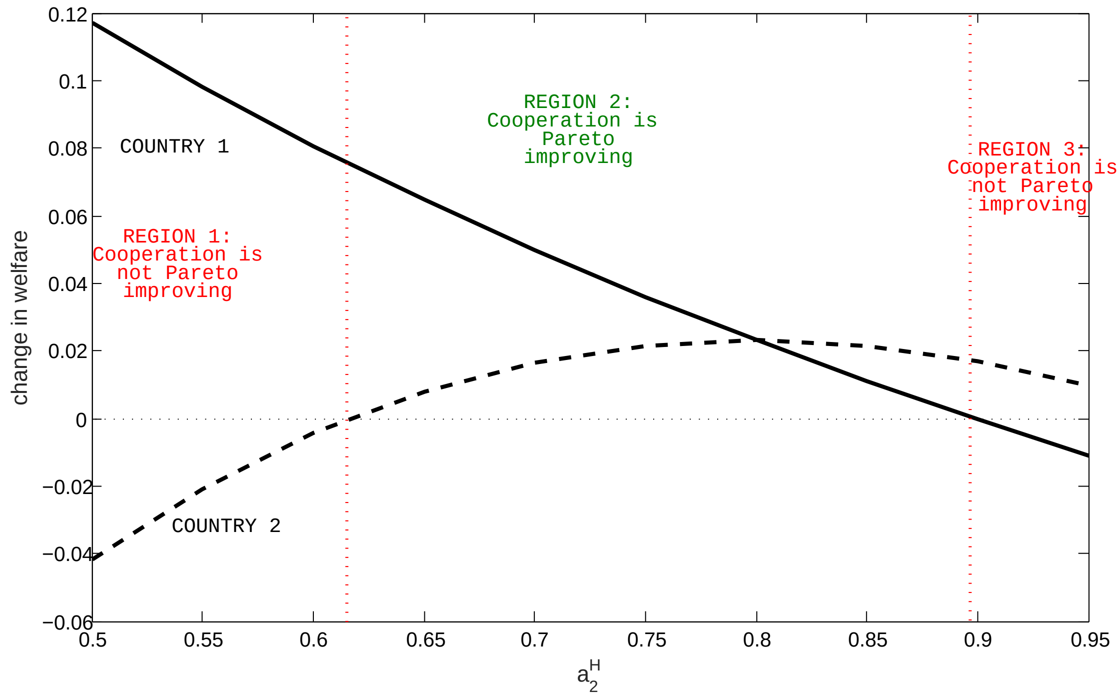
6.1 The welfare consequences of cooperation

Our hypothesis is that central banks cooperate to implement the first-best i.e. the CPO allocation. The strategy of the proof is then to show that $\Delta U_h(s) \equiv U_h^{CPO}(s) - U_h^N(s) > 0$ for every h and s when $z_h \to 0$. Since the result holds for sufficiently small shocks, the type of shock (symmetric or asymmetric) does not really matter.

Proposition 9 *Given a_1^H, all agents are better off under cooperation than under non-cooperation for an open set around $a_2^H = a_1^H$ for $z_h \to 0$.*

Proposition 9 shows that both countries would gain from moving from a non-cooperative to a cooperative environment as long as countries are sufficiently similar. As an illustrative example, we set $a_1^H = 0.8$ and draw ΔU_1 and ΔU_2 as functions of a_2^H in Figure 1. When countries have the same degree of openness, the dotted and the solid lines intersect implying that the countries experience the same welfare gains. However, if the countries are too heterogeneous then one of the two might be worse off. Utility in country 1 (2) falls (increases) as country 2 becomes more closed. If country 2 is less open than country 1, then country 2 gains at the expense of country 1 (Region 3). The reason is that a high weight must be given to country 2 to compensate for the fall in leisure resulting from the high demand for the domestic good. If country 2 is more open than country 1, then the opposite occurs (Region 1).

Figure 1: The welfare consequences of cooperation for $a_1^H = 0.8$



One might be tempted to argue that, after all, cooperation is more likely between countries whose degree of home bias is very similar, such as developed countries. Hence, in reality, cooperating countries would always lie within Region 2, where the first-best allocation is Pareto improving on the Nash equilibrium. However, the degree of home bias is often set in calibration exercises either to 0.8 or 0.9[21]. Therefore, a situation where one of the cooperating countries has a degree of home bias equal to 0.8 while the other one is 0.9 is not unrealistic at all. In that case, the first-best allocation is not Pareto improving (Region 3).

When the two countries belong to either Region 1 or in Region 3, one might then wonder if there are other (second-best) allocations which might make both of them better off. In order to study those allocations, we assume that the two central banks decide to treat agents born in the same country but in different states equally and let the relative weight between the two countries to be the free parameter: $\gamma_h^C(s) = \gamma_h^C$ for every h and

[21]See e.g. Corsetti, Dedola and Leduc, 2010; Pappa, 2004. Moreover, it is common to perform robustness exercises with respect to the consumption home bias.

s. For $z_h \to 0$, the optimal monetary transfers under cooperation are:

$$\begin{aligned} \mu^{1C}{}_{z_h \to 0} &= \frac{1}{a_1^H + \gamma a_2^F} - 1 \\ \mu^{2C}{}_{z_h \to 0} &= \frac{1}{a_2^H + \frac{a_1^F}{\gamma}} - 1 \end{aligned}$$

where $\gamma \equiv \frac{\gamma_2^C}{\gamma_1^C}$ is the weight given to agents in country 2 relatively to agents in country 1. When the relative weight is set so that $\gamma = \frac{a_1^F}{a_2^F}$, we recover the CPO allocation analysed above. Following the same steps as for Proposition 9, we can derive the welfare consequences of cooperation:

$$\begin{aligned} \Delta U_1^C{}_{z_h \to 0} &= -\gamma a_2^F + a_1^H(\log(a_1^H + \gamma a_2^F) - \log(a_1^H)) + \\ &+ a_1^F(\log(a_2^H + \frac{a_1^F}{\gamma}) - \log(a_2^H)) \end{aligned} \tag{28}$$

$$\begin{aligned} \Delta U_2^C{}_{z_h \to 0} &= -\frac{a_1^F}{\gamma} + a_2^H(\log(a_2^H + \frac{a_1^F}{\gamma}) - \log(a_2^H)) + \\ &+ a_2^F(\log(a_1^H + \gamma a_2^F) - \log(a_1^H)) \end{aligned} \tag{29}$$

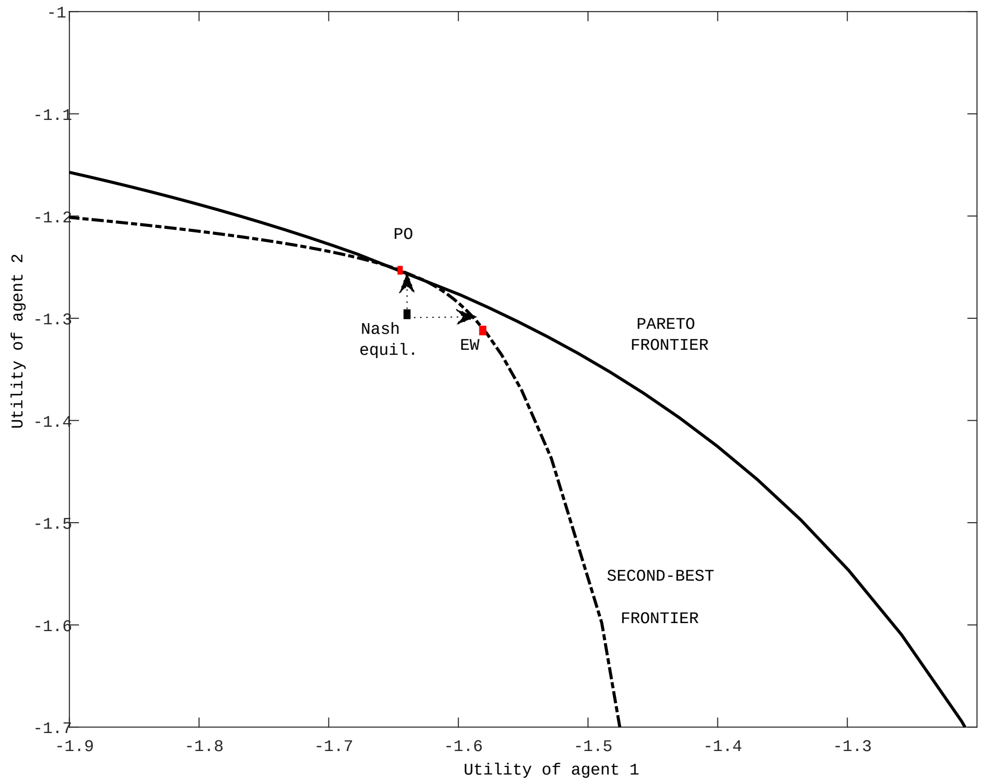
It can be observed that the welfare consequences of moving from the Nash equilibrium to cooperation are generally ambiguous and depend on the degree of home bias of the two countries, as well as on the relative weight.

As an illustration, we then consider the following example: $a_1^H = 0.8$ while $a_2^H = 0.9$. At these values, we already know that country 1 is worse off when the first-best allocation is chosen.

In Figure 2, the solid lines represent the Pareto frontiers of the world economy, while the dotted lines are the second-best frontiers, i.e. the set of all allocations that can be reached through cooperation with the monetary policy instruments. As shown by Proposition 6, only when $\gamma = \frac{\gamma_2}{\gamma_1} = \frac{a_1^F}{a_2^F}$, the two coincides and a first-best allocation can be implemented through cooperation. We also consider the case where the two central banks cooperate but give the same weight to both agents: $\gamma = 1$. The case of a utilitarian social welfare function is indeed standard in the literature (see Corsetti, Dedola and Leduc, 2010). Although this allocation does not belong to the set of CPO allocations, it might still be Pareto improving as it is a second-best solution.

As country 2's degree of openness is lower than country 1's, country 2 loses from cooperation under "equal weights" (EW). As there is a relatively high demand for good 2, agent 2 works more but it is not compensated by having been assigned a higher weight

Figure 2: First-best versus second-best policy when $a_1^H = 0.8$ and $a_2^H = 0.9$

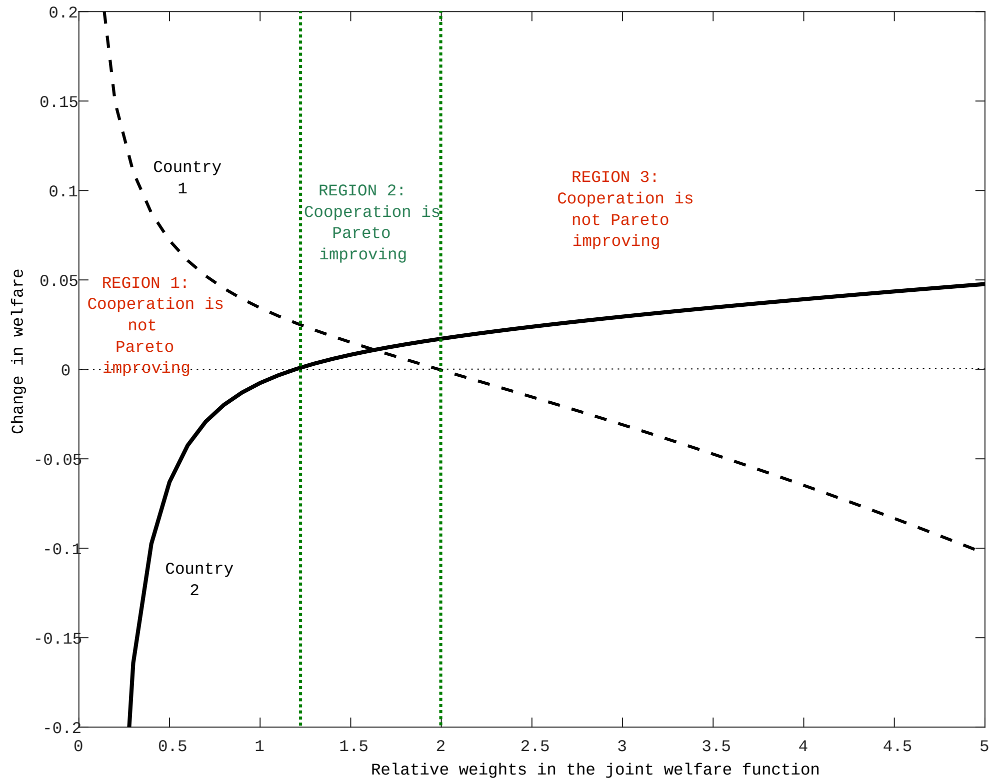


in the welfare function (25). In the CPO allocation, central banks would take into account the countries' heterogeneity and, as a result, they would set $\gamma_1 = \frac{1}{3}$ and $\gamma_2 = \frac{2}{3}$. However, Figure 2 shows that agent 1 would lose from such policy (see also Figure 1).

Although cooperation cannot implement an allocation that is simultaneously CPO *and* Pareto improving on the Nash equilibrium, Figure 2 shows that there is a set of allocations on the second-best frontier which improve on the Nash equilibrium. Figure 3 illustrates the weights which correspond to those allocations. Cooperation between the two central banks would be Pareto improving provided that the relative weight of agent 2 γ is set between 1.3 and 1.9. This implies that agent 2 should be given a weight between 0.56 and 0.66.

Our example shows that even when countries are quite heterogeneous, there exists a set of Pareto improving allocations that can be achieved under cooperation. However, since the two countries should be weighted differently in order to achieve an outcome that is favorable for both, then it is unlikely that the country which is assigned a lower weight agrees to the monetary policies required.

Figure 3: The set of relative weights γ associated with Pareto improvements when $a_1^H = 0.8$ and $a_2^H = 0.9$



Countries' asymmetries can then explain why international monetary policy cooperation is difficult to achieve. Obstfeld and Rogoff (2002) argued instead that the reason behind the lack of cooperation is rather that monetary policy spillovers are a second-order problem as compared to internal objectives such as price and output stabilization[22]. The next section will show that this is not the case in our framework, since the gains from international monetary cooperation are not negligible.

6.2 The gains from cooperation

Proposition 9 indicates that all agents would gain from a regime of central banks' cooperation for an open set of parameters.

We shall now attempt to quantify those gains. In order to do that, we measure the welfare gains from cooperation as the percentage increase in wealth that would give agents under the Nash equilibrium the same level of utility as under cooperation (i.e. Hicksian equivalent variation).

[22] A similar point was recently made by Blanchard (2016).

We then define the welfare of the (global) economy under cooperation and non-cooperation as follows:

$$V^C = \sum_h \sum_s U_h^C(s)$$

$$V^N = \sum_h \sum_s U_h^N(s)$$

where V^C is welfare under cooperation and V^N is welfare under non-cooperation (Nash equilibrium). Therefore, we define as ϕ as the percentage increase in wealth that would yield a level of total welfare under the Nash equilibrium equal to welfare in a cooperative outcome. This implies that:

$$\phi = (\exp(V^C - V^N) - 1) \times 100$$

We set our baseline parameters consistently with the literature (see e.g. Pappa (2004) and Rabitsch (2012)). We quantify the gains both in the case of asymmetric and symmetric shocks, modelling the demand shocks as in Tables 1 and 2.

Table 3: Parameter Values

Home bias	$a^H = 0.8$
Persistence	$p = 0.9$
Inverse of labour elasticity	$\eta = 3$

Both in the case of asymmetric and symmetric shocks, the welfare gains are a non-negligible 0.6% in terms of equivalent consumption. This result is at odds with the New Keynesian literature, which typically finds that the welfare gains from cooperation are extremely small[23]. The comparison is especially striking considering that our utility function is logarithmic. In a New Keynesian setting, the gains from cooperation are exactly zero in this case[24].

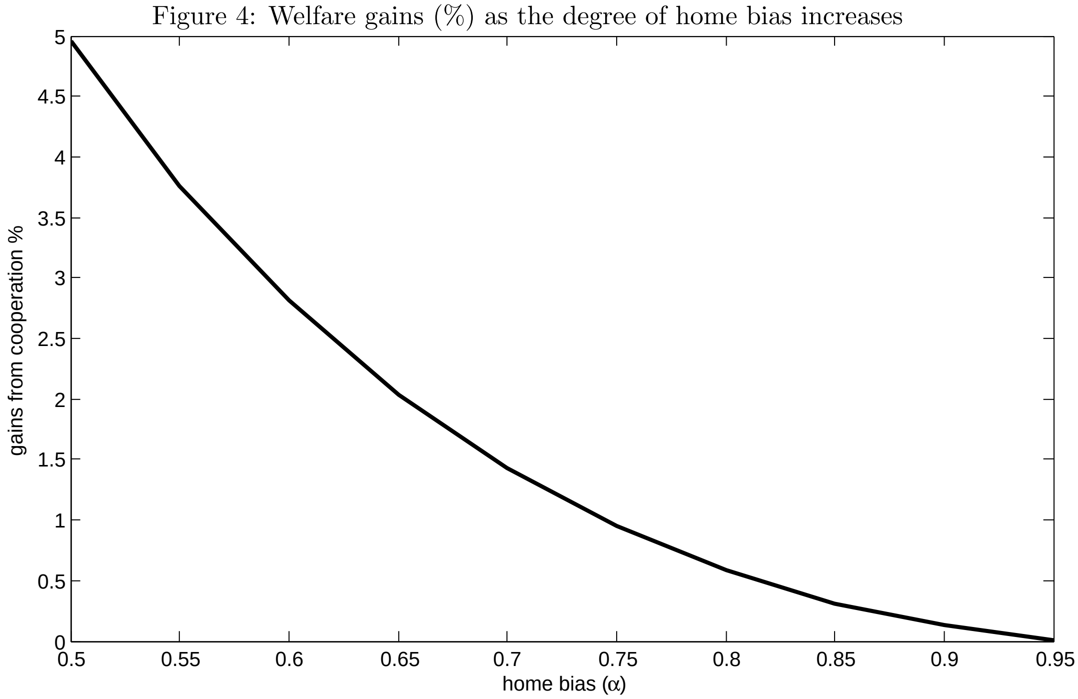
In fact, our paper proposes a entirely different mechanism through which monetary policy can have real effects. The OLG structure implies that changes in money supply have an impact on the agents' budget constraints, since only part of the population receives the monetary transfer from the domestic central bank. Under these circumstances, agents'

[23] See e.g. Obstfeld and Rogoff (2002), Corsetti, Dedola and Leduc (2010), Fujiwara and Wang (2017).

[24] See also Corsetti and Pesenti (2005).

decisions about how much labour to supply are affected by monetary policies, since the expectation to receive a monetary transfer in the future generates a positive wealth effect. On the contrary, let us consider versions of the New Keynesian model where money is modelled explicitly by entering the utility functions of the agent (e.g. Corsetti and Pesenti, 2005). Wealth effects from monetary policy are absent: changes to the government budget constraint do not affect the equilibrium equations. This is due to the representative agent assumption, as well as the assumption that only domestic residents can hold the domestic currency[25].

Figure 4: Welfare gains (%) as the degree of home bias increases



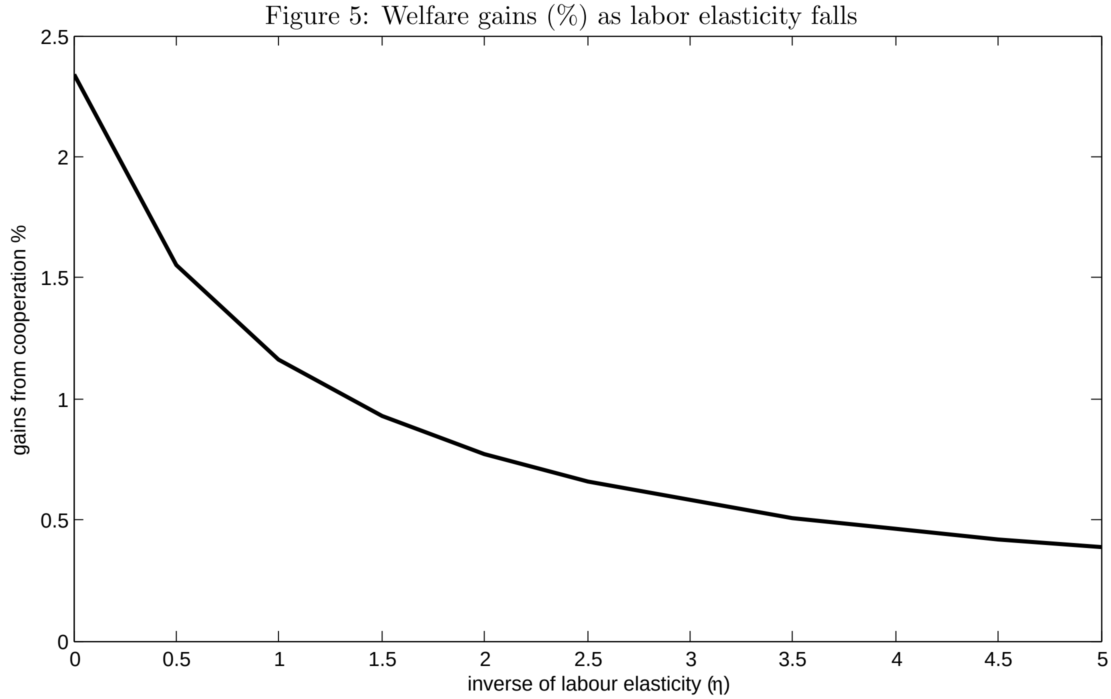
In Figures 4 and 5, we perform some sensitivity analysis with respect to the degree of home bias and the labour elasticity[26].

Figure 4 illustrates that the maximum gains from cooperation can be achieved when agents give the same weight to the domestic and the foreign good (full openness). The welfare gains are almost equal to 5%, therefore they are far from negligible. As the degree of home bias increases, the gains from cooperation fall. If agents' preferences are heavily biased towards the domestic good, then the consumption losses that arise from pursuing

[25] The incentive to deviate from the cooperative solution in the New Keynesian models hinges instead on the possibility to manipulate the terms of trade of the economy. As this effect is wiped out in the logarithmic case, then there are no gains from cooperation when utility is logarithmic. If the elasticity of substitution is different than one, the gains are positive but still extremely small (see e.g. Corsetti, Dedola and Leduc, 2010).

[26] We do the exercise for the case of asymmetric shocks.

Figure 5: Welfare gains (%) as labor elasticity falls



an expansionary monetary policy in the Nash equilibrium are weighted more heavily. Therefore, central banks do not deviate as much from the cooperative outcome and the gains from cooperation are smaller.

Figure 5 shows instead how the gains from cooperation vary with the labour elasticity. Interestingly, the maximum gains from cooperation can be achieved with the labour elasticity approaches infinity ($\eta = 0$). As the labour elasticity falls, central banks' incentive to deviate from the cooperation outcome is smaller as domestic agents react less to the central bank's policy. In fact, it is easy to verify that an increase in the monetary transfer has a smaller effect on labour supply the lower is the elasticity of labour supply (see equation (5)).

7 Conclusions

In this paper, we revisit two central issues for open economies: whether central banks should care about exchange rate volatility and the gains from monetary policy cooperation. Our model is rich of policy implications.

Firstly, we show that the exchange rate volatility that we observe between any two currencies might be far too high. Excess exchange rate volatility is the result of two factors: firstly, it is the outcome of an incomplete markets' environment, since agents are not able to insure against the shocks which are behind exchange rate movements; secondly, central banks' policies raise the volatility of the nominal exchange rate when acting in a non-cooperative environment. Cooperation among central banks would then achieve two things. Firstly, it would use monetary policy to smooth the effects of shocks to economic fundamentals on the exchange rate, implementing policies which "lean against the wind". Secondly, it would curb the excess volatility associated with non-cooperative policies. Although cooperation can effectively "complete the markets" by implementing a first-best allocation, it does not implement a pegged exchange rate or a monetary union. First-best allocations are still characterized by some degree of exchange rate volatility.

This paper also identifies a new channel through which central banks can impose a negative spillover to the rest of the world when setting their monetary policy optimally. Differently from representative agent models, changes in money supply have an effect on the equilibrium allocation despite prices are fully flexible. In a non-cooperative environment, central banks have the incentive to devaluate their domestic currency by printing money supply. By doing so, each of them imposes a negative spillover to the other country in terms of lower consumption for the foreign good. In this context, we show that gains from cooperation exist. For commonly used parameter values, these gains amount to a welfare increase of 0.6% in equivalent consumption. It is important to stress that this is not a negligible number, considering that we use a logarithmic utility function which typically gives zero gains from cooperation in other settings (e.g. see Corsetti, Dedola and Leduc, 2010).

Finally, our model suggests a novel reason for the lack of monetary policy cooperation among central banks. In order for cooperation to be Pareto improving, some countries should be prepared to accept that other countries must carry a higher weight in the social welfare function. Therefore, our analysis implies that a "neutral party", such as the IMF or other multinational institutions, might have a role in trying to bring central banks together to reap the gains from cooperation.

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Appendix

First-order conditions

Decentralized economy

We set up the Lagrangian function to analyze the maximization problem of an agent born in event s^t:

$$\begin{aligned}\mathcal{L}_h(s^t) &= -\frac{l_h(s)^{1+\eta}}{1+\eta} + \sum_{s'} f(s'|s)[a_h^1(s)\log c_h^1(s'|s) + a_h^2(s)\log c_h^2(s'|s)] + \\ &+ \lambda_h(s^t)[\tilde{w}_h(s^t) - \tilde{m}_h^1(s^t) - \varepsilon(s^t)\tilde{m}_h^2(s^t)] + \\ &+ \sum_{s^{t+1}|s^t} \lambda_h(s^{t+1}|s^t)\cdot\left[\frac{\tilde{m}_h(s^t)}{1+\pi(s^{t+1}|s^t)} + \tilde{T}_h(s^{t+1}|s^t) - c_h(s'|s)\right]\end{aligned}$$

The first-order conditions are:

$$l_1(s) \;:\; -l_1(s)^\eta + \lambda_1(s^t)\tilde{\omega}_1(s^t) = 0 \tag{30}$$

$$l_2(s) \;:\; -l_2(s)^\eta + \lambda_2(s^t)\varepsilon(s^t)\tilde{\omega}_2(s^t) = 0 \tag{31}$$

$$c_h^\ell(s'|s) \;:\; f(s'|s)\frac{a_h^\ell(s)}{c_h^\ell(s'|s)} - \lambda_h^\ell(s^{t+1}|s^t) = 0 \qquad \forall\ s' \tag{32}$$

$$\tilde{m}_h^1(s^t) \;:\; -\lambda_h(s^t) + \sum_{s^{t+1}|s^t}\frac{\lambda_h^1(s^{t+1}|s^t)}{1+\pi^1(s^{t+1}|s^t)} = 0 \tag{33}$$

$$\tilde{m}_h^2(s^t) \;:\; -\lambda_h(s^t)\varepsilon(s^t) + \sum_{s^{t+1}|s^t}\frac{\lambda_h^2(s^{t+1}|s^t)}{1+\pi^2(s^{t+1}|s^t)} = 0 \tag{34}$$

$$\lambda_h(s^t) \;:\; \tilde{w}_h(s^t) - \tilde{m}_h^1(s^t) - \varepsilon(s^t)\tilde{m}_h^2(s^t) = 0 \tag{35}$$

$$\lambda_h^\ell(s^{t+1}|s^t) \;:\; \frac{\tilde{m}_h^\ell(s^t)}{1+\pi^\ell(s^{t+1}|s^t)} + \tilde{T}_h^\ell(s^{t+1}|s^t) - c_h^\ell(s'|s) = 0 \qquad \forall\ s^{t+1},\ell \tag{36}$$

Planner's problem

First, we can consolidate (9), (10), (11) and (12) as follows:

$$c_1^1(s|s') + c_2^1(s|s') = Z^1(s)l_1(s) \qquad \forall\ s,s' \tag{37}$$

$$c_1^2(s|s') + c_2^2(s|s') = Z^2(s)l_2(s) \qquad \forall\ s,s' \tag{38}$$

Let $\lambda^1(s|s')$ and $\lambda^2(s|s')$ be the Lagrange multipliers associated with (37) and (38) respectively. The first-order conditions of the problem are:

$$\begin{aligned}
l_1(s) &: -\gamma_1^P(s)l_1(s)^\eta + \sum_{s'} \lambda^1(s|s')Z^1(s) = 0 \qquad \forall\ s && (39)\\
c_1^1(s|s') &: \frac{\gamma_1^P(s')f(s|s')a_1^H(s')}{c_1^1(s|s')} = \lambda^1(s|s') \qquad \forall\ s,s' && (40)\\
c_2^1(s|s') &: \frac{\gamma_2^P(s')f(s|s')a_2^F(s')}{c_2^1(s|s')} = \lambda^1(s|s') \qquad \forall\ s,s' && (41)\\
l_2(s) &: -\gamma_2^P(s)l_2(s)^\eta + \sum_{s'} \lambda^2(s|s')Z^2(s) = 0 \qquad \forall\ s && (42)\\
c_1^2(s|s') &: \frac{\gamma_1^P(s')f(s|s')a_1^F(s')}{c_1^2(s|s')} = \lambda^2(s|s') \qquad \forall\ s,s' && (43)\\
c_2^2(s|s') &: \frac{\gamma_2^P(s')f(s|s')a_2^H(s')}{c_2^2(s|s')} = \lambda^2(s|s') \qquad \forall\ s,s' && (44)\\
\lambda^1(s|s') &: Z^1(s)l_1(s) - c_1^1(s|s') - c_2^1(s|s') = 0 \qquad \forall\ s,s' && (45)\\
\lambda^2(s|s') &: Z^2(s)l_2(s) - c_1^2(s|s') - c_2^2(s|s') = 0 \qquad \forall\ s,s' && (46)
\end{aligned}$$

Combine (40), (41) and (45) to obtain:

$$c_1^1(s|s') = \frac{\gamma_1^P(s')a_1^H(s')}{\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s')} l_1(s)Z^1(s) \tag{47}$$

To solve for $l_1(s)$, plug equation (47) into (40) to get:

$$f(s|s')(\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s')) = \lambda^1(s|s')l_1(s)Z^1(s)$$

Summing across s' and using (39), we find the optimal quantity of labour in country 1:

$$l_1^P(s) = \left[\frac{\sum_{s'} f(s|s')(\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s'))}{\gamma_1^P(s)}\right]^{\frac{1}{1+\eta}}$$

Plugging the last equation into (47) and using the feasibility condition (45), we find the allocation of good 1 between the two agents:

$$\begin{aligned}
c_1^{1P}(s|s') &= \frac{\gamma_1^P(s')a_1^H(s')}{\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s')} l_1^P(s)Z^1(s)\\
c_2^{1P}(s|s') &= \frac{\gamma_2^P(s')a_2^F(s')}{\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s')} l_1^P(s)Z^1(s)
\end{aligned}$$

Following the same steps, we can find the labour supply in country 2 and the consumption allocation for good 2:

$$\begin{aligned}
l_2^P(s) &= \left[\frac{\sum_{s'} f(s|s')(\gamma_2^P(s')a_2^H(s') + \gamma_1^P(s')a_1^F(s'))}{\gamma_2^P(s)}\right]^{\frac{1}{1+\eta}}\\
c_2^{2P}(s|s') &= \frac{\gamma_2^P(s')a_2^H(s')}{\gamma_2^P(s')a_2^H(s') + \gamma_1^P(s')a_1^F(s')} l_2^P(s)Z^2(s)\\
c_1^{2P}(s|s') &= \frac{\gamma_1^P(s')a_1^F(s')}{\gamma_2^P(s')a_2^H(s') + \gamma_1^P(s')a_1^F(s')} l_2^P(s)Z^2(s)
\end{aligned}$$

Proofs

Proof of Proposition 1

We proceed in a few steps to show the result.

Step 1 - Firstly, we show that the real money balances of currency ℓ are equal to domestic output: $\tilde{M}^\ell(s^t) - y^\ell(s) = 0$ for every ℓ. This implies that real money balances are not history dependent $\tilde{M}^\ell(s^t) = \tilde{M}^\ell(s)$.

Using the fact that $\tilde{\omega}(s)l(s) = Z(s)L(s) = y(s)$ holds in every country, and assuming that money markets clear, we can sum across h the budget constraint of the young as follows:

$$\varepsilon(s^t)(\tilde{M}^2(s^t) - y^2(s)) + (\tilde{M}^1(s^t) - y^1(s)) = 0$$

Note that $\varepsilon(s^t)$ is strictly positive. We now argue that, in equilibrium, the real money balances of currency ℓ are equal to output in country ℓ:

$$\tilde{M}^\ell(s^t) - y^\ell(s) = 0 \qquad \ell = 1, 2$$

from which it follows that real money balances are not history dependent:

$$\tilde{M}^\ell(s^t) = \tilde{M}^\ell(s) \qquad \forall\ s^t \tag{48}$$

On the contrary, suppose that $\tilde{M}^\ell(s^t) - y^\ell(s) \neq 0$. Aggregating the budget constraints of the old in event s^t, we have that $p^\ell(s^t)c^\ell(s|s') = (1 + \mu^\ell(s))M^\ell(s^{t-})$. Using equation (1), this can be rewritten as $c^\ell(s|s') = \tilde{M}^\ell(s^t)$. But then, we have by substitution that $c^\ell(s|s') - y^\ell(s) \neq 0$. Therefore, feasibility is violated.

This result allows us to derive the gross rate of inflation for each good:

$$1 + \pi^\ell(s^{t+1}|s^t) \equiv \frac{p^\ell(s^{t+1}|s^t)}{p^\ell(s^t)} = \frac{M^\ell(s^{t+1}|s^t)}{M^\ell(s^t)}\frac{y^\ell(s)}{y^\ell(s')} = (1 + \mu^\ell(s))\frac{y^\ell(s)}{y^\ell(s')} \tag{49}$$

As expected, inflation is increasing in money growth and decreasing in output growth.

Using the government budget constraint (1), we can rewrite the monetary transfers (which only accrue to domestic agents) as follows:

$$\begin{aligned}\tilde{T}^\ell(s^{t+1}|s^t) \equiv \frac{T^\ell(s^{t+1}|s^t)}{p^\ell(s^{t+1}|s^t)} &= \frac{T_h^\ell(s^{t+1}|s^t)}{p^\ell(s^{t+1}|s^t)}\frac{p^\ell(s^t)}{p^\ell(s^t)} = \frac{\mu^\ell(s)\tilde{M}^\ell(s)}{1 + \pi^\ell(s^{t+1}|s^t)} = \\ &= \frac{\mu^\ell(s)y^\ell(s)}{1 + \pi^\ell(s^{t+1}|s^t)} = \frac{\mu^\ell(s)l^\ell(s)Z^\ell(s)}{1 + \pi^\ell(s^{t+1}|s^t)}\end{aligned} \tag{50}$$

Step 2 - From Step 1, we can derive the implication that relative prices do not depend on the history of the economy: $\varepsilon(s^t) = \varepsilon(s)$.

We can combine equations (32), (33) and (34) and obtain the following expression for relative prices in event s^t:

$$\varepsilon(s^t) = \frac{\sum_{s^{t+1}|s^t} \frac{f(s'|s)a_h^2(s)}{c_h^2(s'|s)(1+\pi^2(s^{t+1}|s^t))}}{\sum_{s^{t+1}|s^t} \frac{f(s'|s)a_h^1(s)}{c_h^1(s'|s)(1+\pi^1(s^{t+1}|s^t))}} \tag{51}$$

Plugging the inflation rates (49) into (51), we get:

$$\varepsilon(s^t) = \frac{\sum_{s'|s} \frac{f(s'|s)a_h^2(s)}{c_h^2(s'|s)(1+\mu^2(s))} \frac{y^2(s')}{y^2(s)}}{\sum_{s'|s} \frac{f(s'|s)a_h^1(s)}{c_h^1(s'|s)(1+\mu^1(s))} \frac{y^1(s')}{y^1(s)}} \tag{52}$$

which implies that relative prices depend on the current and future states, but not on the past: $\varepsilon(s^t) = \varepsilon(s)$. We are therefore left with S relative prices as endogenous variables.

Step 3 - Next, we show that only S goods' markets equilibrium equations are independent.

Suppose that the money markets clear for all events. Consider an event s^t with the following characteristics: s is the realized state in that particular event and also in the previous time period. Aggregate consumption can be written as:

$$c^\ell(s|s) = \tilde{M}^\ell(s^t) \tag{53}$$

Consider also another event $s^{t'}$ in which the realized state is s but yesterday's state is instead s'. Aggregate consumption satisfies the following equation:

$$c^\ell(s|s') = \tilde{M}^\ell(s^{t'}) \tag{54}$$

Because of Step 1, we have that $\tilde{M}^\ell(s^t) = \tilde{M}^\ell(s^{t'}) = \tilde{M}^\ell(s)$. Hence, $c^\ell(s|s') = c^\ell(s|s) = \tilde{M}^\ell(s)$. Suppose that the goods markets equations clear if yesterday's state is the same as today:

$$c^\ell(s|s) = y^\ell(s) \qquad\qquad \text{(2S equations)}$$

As the aggregate consumption of any good does not depend on the past, $c^\ell(s|s') = y^\ell(s)$ automatically clear for $s' \neq s$. Therefore, only $2S$ equations in the goods' markets are independent.

Finally, we can get rid of further S equations by Walras Law. Using our previous results, the aggregated budget constraint of the young can be rewritten as:

$$c^1(s|s) - y^1(s) + \varepsilon(s)(c^2(s|s) - y^2(s)) = 0$$

If the markets for, say, good 1 clear, then the markets for good 2 do also.

Step 4 - Finally, we can further solve the maximisation problem of both agents.

Rearranging the first-order conditions for $h = 1$, and using the fact that $\tilde{\omega}_1(s^t) = Z^1(s)$ and $\tilde{T}^1(s^{t+1}|s^t) = \frac{\mu^1(s)l^1(s)Z^1(s)}{1+\pi^1(s^{t+1}|s^t)}$, we get that:

$$\begin{aligned} \tilde{m}_1^1(s^t) &= \frac{a_1^1(s)Z^1(s)}{l_1(s)^\eta} - \mu^1(s)l^1(s)Z^1(s) \\ \tilde{m}_1^2(s^t) &= \frac{a_1^2(s)Z^1(s)}{l_1(s)^\eta \varepsilon(s)} \end{aligned}$$

As intuition suggests, the demand for the domestic currency is negatively related to the size of the monetary transfer that the agent expects to receive next period from the domestic central bank. On the other hand, the demand for the foreign currency only depends on wealth and on the weight attached to the foreign good.

Plugging the last two equations into (3), we can solve for the labour supply of agent 1:

$$l_1(s) = \frac{1}{(1+\mu^1(s))^{\frac{1}{1+\eta}}}$$

The demand for the two currencies of agents born in country 1 is then:

$$\begin{aligned} \tilde{m}_1^1(s^t) &= \frac{Z^1(s)}{(1+\mu^1(s))^{\frac{1}{1+\eta}}}(a_1^1(s) - a_1^2(s)\mu^1(s)) \\ \tilde{m}_1^2(s^t) &= \frac{a_1^2 Z^1(s)}{\varepsilon(s)}(1+\mu^1(s))^{\frac{\eta}{1+\eta}} \end{aligned}$$

where $\frac{a_1^1(s)}{a_1^2(s)} > \mu^1(s)$[27].

Similarly, we can solve for the maximisation problem of agent 2 and obtain his labour supply:

$$l_2(s) = \frac{1}{(1+\mu^2(s))^{\frac{1}{1+\eta}}}$$

as well as his demand for real money balances:

$$\begin{aligned} \tilde{m}_2^1(s^t) &= a_2^1(s)\varepsilon(s)Z^2(s)(1+\mu^2(s))^{\frac{\eta}{1+\eta}} \\ \tilde{m}_2^2(s^t) &= \frac{Z^2(s)}{(1+\mu^2(s))^{\frac{1}{1+\eta}}}(a_2^2(s) - a_2^1(s)\mu^2(s)) \end{aligned}$$

[27]Notice that the demand for the domestic good is typically no less than the demand for the foreign good: $\frac{a_1^1(s)}{a_1^2(s)} > 1$. Moreover, we have assumed that $\mu^\ell(s) < 1$. Therefore, the demand for the domestic currency is positive. The same will be true for agents born in country 2

Using the budget constraints, we can derive the demand for the two goods of the two agents:

$$
\begin{aligned}
c_1^1(s'|s) &= \frac{a_1^1(s)Z^1(s')}{(1+\mu^1(s'))^{\frac{1}{1+\eta}}} \\
c_1^2(s'|s) &= \frac{a_1^2(s)Z^1(s)Z^2(s')}{\varepsilon(s)Z^2(s)}\frac{(1+\mu^2(s))^{\frac{-\eta}{1+\eta}}}{(1+\mu^2(s'))^{\frac{1}{1+\eta}}}(1+\mu^1(s))^{\frac{\eta}{1+\eta}} \\
c_2^1(s'|s) &= \frac{a_2^1(s)\varepsilon(s)Z^2(s)Z^1(s')}{Z^1(s)}\frac{(1+\mu^1(s)^{\frac{-\eta}{1+\eta}})}{(1+\mu^1(s'))^{\frac{1}{1+\eta}}}(1+\mu^2(s))^{\frac{\eta}{1+\eta}} \\
c_2^2(s'|s) &= \frac{a_2^2(s)Z^2(s')}{(1+\mu^2(s'))^{\frac{1}{1+\eta}}}
\end{aligned}
$$

Firstly, notice that the demand for real money balances is independent from history: $\tilde{m}_h^\ell(s^t) = \tilde{m}_h^\ell(s)$. Therefore, the monetary equations reduce to:

$$
\sum_h \tilde{m}_h^\ell(s) = \tilde{M}^\ell(s) \qquad \forall\ \ell, s
$$

Given the above, we can find the equilibrium value of $\varepsilon(s)$ by plugging the demand functions for good 1 into the market clearing equation:

$$
\sum_h c_h^1(s'|s) = y^1(s')
$$

The equilibrium terms of trade are equal to:

$$
\varepsilon(s) = \frac{a_1^2(s)}{a_2^1(s)}\frac{Z^1(s)}{Z^2(s)}\left(\frac{1+\mu^1(s)}{1+\mu^2(s)}\right)^{\frac{\eta}{1+\eta}}
$$

Using the solution for $\varepsilon(s)$, we can find the solution for consumption.

Since $y^\ell(s) = \tilde{M}^\ell(s)$, we have seen that we can easily calculate inflation rates between any two time periods and for any history of endowments using equation (49). The only thing left to determine is the initial prices. At the initial node s^0, one of the S states are realized and the real money balances that the initial old hold depend on that initial state. Denoting initial prices as $p^\ell(s^0)$, we then obtain their solutions as follows:

$$
p^\ell(s^0) = \frac{\bar{M}^\ell(1+\mu^\ell(s))^{\frac{1}{1+\eta}}}{Z^\ell(s)}
$$

where $\bar{M}^\ell$ is the initial money supplies.

Proof of Proposition 2

When $\mu^1(s) = \mu^2(s) = 0$, the competitive equilibrium of the economy is (see equations (1):

$$\begin{array}{ll} l_1^{CE}(s) = 1 & l_2^{CE}(s) = 1 \\ c_1^{1CE}(s|s') = a_1^H(s')Z^1(s) & c_1^{2CE}(s|s') = a_2^F(s')Z^2(s) \\ c_2^{1CE}(s|s') = a_1^F(s')Z^1(s) & c_2^{2CE}(s|s') = a_2^H(s')Z^2(s) \end{array} \tag{55}$$

Suppose that the competitive equilibrium is CPO: $l_h^{CE}(s) = l_h^P(s)$ and $c_h^{\ell CE}(s|s') = c_h^{\ell P}(s|s')$ for every h and ℓ. Then, the following conditions on the planner's weights must hold:

$$\gamma_1^P(s) = \sum_{s'} f(s|s')\gamma_1^P(s')a_1^H(s') + \gamma_2^P(s')a_2^F(s') \tag{56}$$

$$\gamma_2^P(s) = \sum_{s'} f(s|s')\gamma_2^P(s')a_2^H(s') + \gamma_1^P(s')a_1^F(s') \tag{57}$$

$$\gamma_1^P(s')a_1^F(s') = \gamma_2^P(s')a_2^F(s') \tag{58}$$

Condition (58) implies that (56) and (57) can be rewritten as:

$$\gamma_1^P(s) = \sum_{s'} f(s|s')\gamma_1^P(s') \tag{59}$$

$$\gamma_2^P(s) = \sum_{s'} f(s|s')\gamma_2^P(s') \tag{60}$$

Using condition (58), let us define $A(s) \equiv \frac{\gamma_1^P(s)}{\gamma_2^P(s)} = \frac{a_2^F(s)}{a_1^F(s)}$. Exploiting this definition, we can rewrite (59) as follows:

$$\gamma_2^P(s)A(s) = \sum_{s'} f(s|s')\gamma_2^P(s')A(s') \tag{61}$$

But then, the latter equation and condition (60) imply that:

$$\sum_{s'} f(s|s')\gamma_2^P(s') = \frac{\sum_{s'} f(s|s')\gamma_2^P(s')A(s')}{A(s)}$$

or

$$\sum_{s'} f(s|s')\gamma_2^P(s')\left(1 - \frac{A(s')}{A(s)}\right) = 0$$

Since $f(s|s') > 0$ and $\gamma_2^P(s) > 0$ for every s and s', for all conditions to hold it must be that $A(s') = A(s) = A$ for every s and s'.

On the other hand, suppose that $\frac{a_2^F(s)}{a_1^F(s)} = A$ for every s. Then, the competitive equilibrium is Pareto optimal as there exists a vector of weights (58), (59) and (60) that supports such allocation.

Proof of Proposition 3

Each central bank chooses the vector of monetary transfers that maximises their welfare function subject to the equilibrium restrictions, respectively (19) and (20) subject to (21) and (22). In doing so, each takes the actions of the other central bank as given.

Let us consider the welfare maximization problem of the central bank of country 1. The first-order conditions are:

$$\mu^1(s) \ : \ \frac{\gamma_1^N(s)}{(1+\mu^1(s))^2} - \frac{1}{1+\mu^1(s)} \sum_{s'} f(s|s')\gamma_1^N(s')a_1^H(s') = 0 \qquad \forall \ s$$

Notice that an increase in the monetary transfer in state s increases the leisure of the young born in that state. However, it has a negative impact on the utility of all agents when state s is realized when old because output, and hence consumption, falls as a result of the decision of the agent born in state s to supply less labour. The above equation can be rearranged as:

$$\mu^{1N}(s) = \frac{\gamma_1^N(s)}{\sum_{s'} f(s|s')\gamma_1^N(s')a_1^H(s')} - 1 \qquad \forall \ s$$

This is a dominant strategy for the central bank of country 1, as the optimal vector of transfers does not depend on the actions of the other central bank. We leave the calculation of the welfare maximisation problem in country 2 to the reader, as it is similar. If both central banks play their dominant strategy, no central bank has the incentive to deviate. Therefore, $(\mu^{1N}(s), \mu^{2N}(s))$ is a Nash equilibrium of the monetary policy game between the two central banks.

Proof of Proposition 4

The first-order conditions of the cooperation problem are:

$$\begin{aligned}
\mu^1(s) \quad &: \quad \frac{\gamma_1^C(s)}{(1+\mu^1(s))^2} - \frac{1}{1+\mu^1(s)}\left[\sum_{s'} f(s|s')[\gamma_1^C(s')a_1^H(s') + \gamma_2^C(s')a_2^F(s')\right] = 0 \qquad \forall \ s \\
\mu^2(s) \quad &: \quad \frac{\gamma_1^C(s)}{(1+\mu^2(s))^2} - \frac{1}{1+\mu^2(s)}\left[\sum_{s'} f(s|s')[\gamma_1^C(s')a_1^F(s') + \gamma_2^C(s')a_2^H(s)]\right] = 0 \qquad \forall \ s
\end{aligned}$$

Rearranging the first-order conditions, we find that the optimal transfers are (26) and (27).

Proof of Proposition 5

In order to appropriately compare the Nash allocations and the CPO allocations, we assume that the planner chooses the same Nash weights except for attributing arbitrary weights to the two countries:

$$\gamma_h^P(s) = \theta_h \gamma_h^N(s) \qquad \text{where } \sum_h \theta_h = 1$$

Then:

$$\begin{aligned}
l_1^P(s) &= \frac{\sum_{s'} f(s|s')(\gamma_1^N(s')a_1^H(s') + \frac{\theta_2}{\theta_1}\gamma_2^N(s')a_2^F(s'))}{\gamma_1^N(s)} \\
l_2^P(s) &= \frac{\sum_{s'} f(s|s')(\gamma_2^N(s')a_2^H(s') + \frac{\theta_1}{\theta_2}\gamma_1^N(s')a_1^F(s'))}{\gamma_2^N(s)}
\end{aligned}$$

Given the optimal monetary transfers (23) (24), the labour supplies in the Nash equilibrium are:

$$\begin{aligned}
l_1^N(s) &= \frac{\sum_{s'} f(s|s')\gamma_1^N(s')a_1^H(s')}{\gamma_1^N(s)} \\
l_2^N(s) &= \frac{\sum_{s'} f(s|s')\gamma_2^N(s')a_2^H(s')}{\gamma_2^N(s)}
\end{aligned}$$

it is easy to see that $l_h^P(s) \neq l_h^N(s)$ for $0 < \theta_h < \infty$. Since this implies that $U_h^N(s) \neq U_h^P(s)$ for every s and h for any possible weights, then the Nash equilibrium is suboptimal.

Proof of Proposition 6

Plug the optimal monetary transfers (26) and (27) into the equilibrium conditions (21) and (22) to obtain:

$$\begin{aligned}
l_1^C(s) &= \frac{\sum_{s'} f(s|s')[\gamma_1^C(s')a_1^H(s') + \gamma_2^C(s')a_2^F(s')]}{\gamma_1^C(s)} \\
c_1^{1C}(s|s') &= a_1^H(s')l_1^C(s)Z^1(s) \\
c_2^{1C}(s|s') &= a_1^F(s')l_1^C(s)Z^1(s) \\
l_2^C(s) &= \frac{\sum_{s'} f(s|s')[\gamma_2^C(s')a_2^H(s') + \gamma_1^C(s')a_1^F(s')]}{\gamma_2^C(s)} \\
c_2^{2C}(s|s') &= a_2^H(s')l_2^C(s)Z^2(s) \\
c_1^{2C}(s|s') &= a_2^F(s')l_2^C(s)Z^2(s)
\end{aligned} \tag{62}$$

Suppose that the same weights of the planner are chosen: $\gamma_1^C(s') = \gamma_1^P(s')$ and $\gamma_2^C(s') = \gamma_2^P(s')$. Notice that $l_h^C(s) = l_h^P(s)$ for every h and s. However, $c_h^{\ell C}(s|s') = c_h^{\ell P}(s|s')$ for every h and ℓ only as long as the weights satisfy the condition that $\gamma_1^C(s')a_1^F(s') = \gamma_2^C(s')a_2^F(s')$.

Proof of Proposition 7

When central banks are inactive the nominal exchange rate would take the following values:

$$\begin{aligned}\tilde{e}_D(1) &= \frac{1-a_1^H-z_1}{1-a_2^H+z_2}\\ \tilde{e}_D(2) &= \frac{1-a_1^H+z_1}{1-a_2^H-z_2}\end{aligned}$$

Therefore, it follows that $\tilde{e}_D(2) > \tilde{e}_D(1)$.

Secondly, let us calculate the optimal monetary transfers under non-cooperation and cooperation. Using equations (23), (24), (26) and (27):

$$\begin{array}{ll}\mu^{1N}(1) = \frac{1}{a_1^H - z_1(2p-1)} - 1 & \mu^{1CPO}(1) = 0 \\ \mu^{1N}(2) = \frac{1}{a_1^H + z_1(2p-1)} - 1 & \mu^{1CPO}(2) = 0 \\ \mu^{2N}(1) = \frac{1}{a_2^H + z_2(2p-1)} - 1 & \mu^{2CPO}(1) = \frac{\frac{1-a_1^H-z_1}{1-a_2^H+z_2}}{p\frac{1-a_1^H-z_1}{1-a_2^H+z_2}+(1-p)\frac{1-a_1^H+z_1}{1-a_2^H-z_2}} - 1 \\ \mu^{2N}(2) = \frac{1}{a_2^H - z_2(2p-1)} - 1 & \mu^{2CPO}(2) = \frac{\frac{1-a_1^H+z_1}{1-a_2^H-z_2}}{p\frac{1-a_1^H+z_1}{1-a_2^H-z_2}+(1-p)\frac{1-a_1^H-z_1}{1-a_2^H+z_2}} - 1\end{array} \tag{63}$$

In order to compare the variance under different policy regimes, it is easy to show (working out the definition of variance) that it is enough to compare the spread between the two values of the nominal exchange rate across the three regimes and prove that:

$$\tilde{e}_N(2) - \tilde{e}_N(1) > \tilde{e}_D(2) - \tilde{e}_D(1) > \tilde{e}_{CPO}(2) - \tilde{e}_{CPO}(1)$$

Firstly, let us show that $\tilde{e}_D(2) - \tilde{e}_D(1) > \tilde{e}_{CPO}(2) - \tilde{e}_{CPO}(1)$. Given (7) and (63), the nominal exchange rate under cooperation behaves as follows:

$$\begin{aligned}\tilde{e}_{CPO}(1) &= p\frac{1-a_1^H-z_1}{1-a_2^H+z_2} + (1-p)\frac{1-a_1^H+z_1}{1-a_2^H-z_2}\\ \tilde{e}_{CPO}(2) &= p\frac{1-a_1^H+z_1}{1-a_2^H-z_2} + (1-p)\frac{1-a_1^H-z_1}{1-a_2^H+z_2}\end{aligned}$$

It is easy to see that the exchange rate under cooperation in any state is just a linear combination of the two realisations of the nominal exchange rate that occur when central banks are inactive.

Secondly, we prove that $\tilde{e}_N(2) - \tilde{e}_N(1) > \tilde{e}_D(2) - \tilde{e}_D(1)$. Given (63), the nominal exchange rate takes the following two values in the Nash equilibrium:

$$\begin{aligned}
\tilde{e}_N(1) &= \frac{1-a_1^H-z_1}{1-a_2^H+z_2}\frac{a_2^H-z_2(2p-1)}{a_1^H+z_1(2p-1)} \\
\tilde{e}_N(2) &= \frac{1-a_1^H+z_1}{1-a_2^H-z_2}\frac{a_2^H+z_2(2p-1)}{a_1^H-z_1(2p-1)}
\end{aligned}$$

Next, let us introduce the following definitions:

$$\begin{aligned}
A &\equiv \frac{1-a_1^H+z_1}{1-a_2^H-z_2} \\
B &\equiv \frac{1-a_1^H-z_1}{1-a_2^H+z_2} \\
C &\equiv \frac{a_2^H+z_2(2p-1)}{a_1^H-z_1(2p-1)} \\
D &\equiv \frac{a_2^H-z_2(2p-1)}{a_1^H+z_1(2p-1)}
\end{aligned}$$

Since $A > B$ and $C > D$, it follows that $AC - BD > A - B$ which means that $\tilde{e}_N(2) - \tilde{e}_N(1) > \tilde{e}_D(2) - \tilde{e}_D(1)$.

Proof of Proposition 8

The optimal monetary transfers under non-cooperation and cooperation when shocks are symmetric are:

$$\begin{array}{ll}
\mu^{1N}(1) = \frac{1}{a_1^H-z_1(2p-1)} - 1 & \mu^{1CPO}(1) = 0 \\
\mu^{1N}(2) = \frac{1}{a_1^H+z_1(2p-1)} - 1 & \mu^{1CPO}(2) = 0 \\
\mu^{2N}(1) = \frac{1}{a_2^H-z_2(2p-1)} - 1 & \mu^{2CPO}(1) = \frac{\frac{1-a_1^H-z_1}{1-a_2^H-z_2}}{p\frac{1-a_1^H-z_1}{1-a_2^H-z_2}+(1-p)\frac{1-a_1^H+z_1}{1-a_2^H+z_2}} - 1 \\
\mu^{2N}(2) = \frac{1}{a_2^H+z_2(2p-1)} - 1 & \mu^{2CPO}(2) = \frac{\frac{1-a_1^H+z_1}{1-a_2^H+z_2}}{p\frac{1-a_1^H+z_1}{1-a_2^H+z_2}+(1-p)\frac{1-a_1^H-z_1}{1-a_2^H-z_2}} - 1
\end{array} \tag{64}$$

Then, the nominal exchange rate takes the following values under the three scenarios:

$$\begin{aligned}
\tilde{e}_D(1) &= \frac{1-a_1^H-z_1}{1-a_2^H-z_2} \\
\tilde{e}_D(2) &= \frac{1-a_1^H+z_1}{1-a_2^H+z_2} \\
\tilde{e}_N(1) &= \frac{1-a_1^H-z_1}{1-a_2^H-z_2}\frac{a_2^H+z_2(2p-1)}{a_1^H+z_1(2p-1)} \\
\tilde{e}_N(2) &= \frac{1-a_1^H+z_1}{1-a_2^H+z_2}\frac{a_2^H-z_2(2p-1)}{a_1^H-z_1(2p-1)} \\
\tilde{e}_{CPO}(1) &= p\frac{1-a_1^H-z_1}{1-a_2^H-z_2}+(1-p)\frac{1-a_1^H+z_1}{1-a_2^H+z_2} \\
\tilde{e}_{CPO}(2) &= p\frac{1-a_1^H+z_1}{1-a_2^H+z_2}+(1-p)\frac{1-a_1^H-z_1}{1-a_2^H-z_2}
\end{aligned}$$

Without loss of generality, let us assume that $\tilde{e}_D(2) > \tilde{e}_D(1)$. Solving the inequality, this implies that $z_1(1-a_2^H) > z_2(1-a_1^H)$.

As above, it is easy to show that $\tilde{e}_D(2) - \tilde{e}_D(1) > \tilde{e}_{CPO}(2) - \tilde{e}_{CPO}(1)$, since the two central banks would just "smooth the demand shocks away".

Secondly, we need to prove that $\tilde{e}_N(2) - \tilde{e}_N(1) > \tilde{e}_D(2) - \tilde{e}_D(1)$. Working out the inequality, after a few steps we get that:

$$\frac{1-a_1^H+z_1}{1-a_2^H+z_2}\left(\frac{a_2^H-a_1^H+(z_1-z_2)(2p-1)}{a_1^H-z_1(2p-1)}\right) > \frac{1-a_1^H-z_1}{1-a_2^H-z_2}\left(\frac{a_2^H-a_1^H-(z_1-z_2)(2p-1)}{a_1^H+z_1(2p-1)}\right)$$

Next, let us introduce the following definitions:

$$\begin{aligned}
A &\equiv \frac{1-a_1^H+z_1}{1-a_2^H+z_2} \\
B &\equiv \frac{1-a_1^H-z_1}{1-a_2^H-z_2} \\
C &\equiv \frac{a_2^H-a_1^H+(z_1-z_2)(2p-1)}{a_1^H-z_1(2p-1)} \\
D &\equiv \frac{a_2^H-a_1^H-(z_1-z_2)(2p-1)}{a_1^H+z_1(2p-1)}
\end{aligned}$$

It is easy to show that $C > D$. Since $A > B$, it follows that $AC - BD > A - B$ which means that $\tilde{e}_N(2) - \tilde{e}_N(1) > \tilde{e}_D(2) - \tilde{e}_D(1)$.

Proof of Proposition 9

Notice that for $z_h \to 0$, the optimal monetary transfers when shocks are asymmetric (63) and symmetric (64) both converge to:

$$\begin{array}{ll} \mu^{1N}(1) = \mu^{1N}(2) = \frac{1}{a_1^H} - 1 & \mu^{1CPO}(1) = \mu^{1CPO}(2) = 0 \\ \mu^{2N}(1) = \mu^{2N}(2) = \frac{1}{a_2^H} - 1 & \mu^{2CPO}(1) = \mu^{2CPO}(2) = 0 \end{array} \tag{65}$$

Substituting these into the equilibrium equations (**??**) and then into the utility functions of all agents, we obtain the gains of moving from the Nash equilibrium to a CPO allocation:

$$\begin{aligned} \Delta U_{1\lim_{z_h \to 0}} &\equiv U_1^{CPO}{}_{\lim_{z_h\to 0}} - U_1^N{}_{\lim_{z_h \to 0}} = -1 + a_1^H - a_1^H \log a_1^H - (1 - a_1^H) \log a_2^H \\ \Delta U_{2\lim_{z_h \to 0}} &\equiv U_2^{CPO}{}_{\lim_{z_h\to 0}} - U_2^N{}_{\lim_{z_h \to 0}} = -1 + a_2^H - a_2^H \log a_2^H - (1 - a_2^H) \log a_1^H \end{aligned}$$

Firstly, let us show that $\Delta U_1 = \Delta U_2 > 0$ when $a_2^H = a_1^H = a^H$. When countries are identical, the above equations can be rewritten as:

$$\Delta U_1 = \Delta U_2 = -1 + a^H - \log a^H$$

In this case, it is easy to verify that ΔU is monotonically decreasing in a^H:$\frac{d\Delta U}{da^H} = 1 - \frac{1}{a^H} < 0$. As a^H tends to zero, we have that $\lim_{a^H \to 0} \Delta U = +\infty$. On the other hand, $\lim_{a^H \to 1} \Delta U = 0$. Since $0 \leq a^H \leq 1$, the gains from cooperation are always positive for both agents.

The next step is to prove that there is an open set around $a_1^H = a_2^H = a^H$ where both countries would gain from cooperation. To start with, observe that ΔU_1 and ΔU_2 are functions of two parameters: a_1^H and a_2^H. Our approach is to study each of them as a function of a_2^H, while showing that the properties of each function are invariant with a_1^H.

Let us start with ΔU_1. Firstly, let us calculate the derivative of ΔU_1 with respect to a_2^H: $\frac{\partial \Delta U_1}{\partial a_2^H} = -\frac{1-a_1^H}{a_2^H} < 0$. Secondly, notice that $\Delta U_{1_{a_2^H=0}} = +\infty$ while $\Delta U_{1_{a_2^H=1}} = -1 + a_1^H - a_1^H \log a_1^H$. Let us then calculate the sign of $\Delta U_{1_{a_2^H=1}}$. When $a_1^H = 0$, then $\Delta U_{1_{a_2^H=1, a_1^H=0}} = -1$. On the other hand, $\Delta U_{1_{a_2^H=1, a_1^H=1}} = 0$. It is easy to check that $\Delta U_{1_{a_2^H=1}}$ is increasing in a_1^H. Hence, we can say that $\Delta U_{1_{a_2^H=1}} < 0$ for $a_1^H \leq 1$. Hence, the gains for country 1 are positive but decreasing as a_2^H increases and become negative for some value of a_2^H.

We now study ΔU_2 as a function of a_2^H. Let us calculate the derivative of ΔU_2 with respect to a_2^H: $\frac{\partial \Delta U_2}{\partial a_2^H} = -\log a_2^H + \log a_1^H$. The derivative is zero when $a_1^H = a_2^H$. It can be verified that this point is a maximum since the second derivative is negative. When

$a_1^H = a_2^H$, we already know that $\Delta U_2 > 0$. Hence, there is an open set around $a_1^H = a_2^H$ such that $\Delta U_2 > 0$.

The properties of both functions hold for any a_1^H. Therefore, we have proved that all agents would gain from cooperation for an open set around $a_1^H = a_2^H$.