

**Department of Economics & Finance**

# **Nominal Exchange Rates and Net Foreign Assets' Dynamics: the Stabilization Role of Valuation Effects**

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# Nominal Exchange Rates and Net Foreign Assets' Dynamics: the Stabilization Role of Valuation Effects

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## Abstract

This paper proposes a theory of nominal exchange rate determination to shed light on its role in countries' portfolio choices and its impact on the dynamics of net foreign assets through valuation effects. The model can rationalize the behavior of the US external position over the past 20 years, which has been characterized by persistent current account deficits and stabilizing valuation effects, as a consequence of the increase in emerging market countries' share of world GDP. We also show quantitatively that the valuation channel is a key component of the process of external adjustment, consistently with the empirical literature.

*Keywords:* nominal exchange rate determination, valuation effects, endogenous portfolio choice, net foreign assets' dynamics, incomplete markets, overlapping-generations economies

*JEL classification:* F31, F32, F36, F41

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# 1 Introduction

One of the most relevant developments that characterize the global economy of the recent decades is the rising importance of the so called “valuation channel” in accounting for the dynamics of net foreign assets of many countries<sup>1</sup>. Since the early 1990s, cross-border holdings of assets and liabilities have substantially increased and the traditional method of computing the net foreign assets position of a country, which relied on the cumulation of current account balances over time, has proved to be inaccurate<sup>2</sup>. As the balance of payments does not record changes in the value of foreign assets and liabilities which can arise due to fluctuations of nominal exchange rates and asset prices, it does not reflect the fact that the valuation channel is becoming more important.

For instance, Figure 1 shows the discrepancy between the cumulated current account balances and the net foreign assets position of the United States. According to the former measure, the net foreign assets position of the United States amounted to almost  $-60\%$  of GDP in 2010. However, direct estimates of net foreign assets and liabilities suggest that the net external position was much lower and equal to around  $-20\%$  of GDP. This implies that the US have experienced a substantial wealth transfer from the rest of the world over the past 20 years as the value of their foreign assets has risen relatively to the value of their foreign liabilities. The significance of the valuation channel is not specific to the US. It is interesting to observe that emerging countries in East Asia have faced exactly the opposite situation: while their net external positions have considerably improved over the past decades because of current account surpluses, they have experienced negative valuation effects (Figure 2)<sup>3</sup>. In all the countries considered in Figures 1 and 2, valuation effects have had a stabilizing effect on the net foreign assets position.

The hypothesis of this paper, which is supported by the empirical literature, is that part of these valuation effects can be attributed to fluctuations of nominal exchange rates<sup>4</sup>.

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<sup>1</sup>See Gourinchas et al. (2015) for a recent survey of the literature.

<sup>2</sup>Lane and Milesi-Ferretti (2001, 2007) constructed estimates of foreign assets and liabilities for 145 countries for the period 1970-2011 and were among the first to notice the mismatch between the stock measures (at current prices) and the balance of payments data.

<sup>3</sup>Gourinchas et al. (2015) make similar observations for other emerging countries.

<sup>4</sup>Lane et al. (2010) and Benetrix et al. (2015) built a large database to document countries’ currency exposures and found that exchange-rate driven valuation effects account for a significant fraction of the overall valuation effects. Another source of valuation effects is fluctuations of asset prices, which lead to capital gains and losses on foreign assets and liabilities. Since there is a relatively large literature on this channel, this paper will focus on modelling exchange-rate driven valuation effects. E.g. see Pavlova et al. (2007, 2010), Tille et al. (2010), Devereux et al. (2010), Nguyen (2011), Ghironi et al. (2015) and Stepanchuk et al. (2015).

Gourinchas et al. (2007) have shown that a significant part of the US cyclical external imbalances are eliminated via predictable movements in nominal exchange rates. The exchange rate, through the valuation channel, has therefore a crucial role in the adjustment of the US net external position along with the traditional trade balance channel.

It is well known that the way in which the valuation channel operates depends on the currency composition of a country's foreign assets and liabilities. The United States tend to issue foreign liabilities in dollars while the majority of their foreign assets are denominated in foreign currencies: as a result, the positive valuation effects experienced over the past twenty years should be associated with a depreciation of the dollar against the currencies of the main US lenders. Indeed, Figure 3 shows the steady depreciation of the dollar against the currencies of East Asian, emerging economies, especially since 2004<sup>5</sup>. Although the valuation channel is empirically relevant for many countries, Gourinchas et al. (2015) have recently stressed that designing models able to generate meaningful expected valuation effects has proved to be a challenge.

This paper takes up this task and proposes a two-country model of nominal exchange rate determination and endogenous portfolio choice to shed light on the role of the nominal exchange rate in countries' portfolio choices as well as its impact on the dynamics of net external positions through valuation effects.

The novelty of this model is that both portfolio allocations and the nominal exchange rate are endogenous and can be uniquely determined in equilibrium. As a consequence, the patterns in the data can be explained purely as the result of changes in the fundamentals of the two economies<sup>6</sup>. In particular, the idea behind this paper is that the stylized facts presented in Figures 1-3 can be explained as the result of the rise of emerging market countries in the world economy, as measured by their share of world GDP. In fact, Figure 4 shows the steady increase in the share of world GDP of East Asian countries relatively to the US over the past twenty years. As we would expect, China accounts for a huge part of this trend.

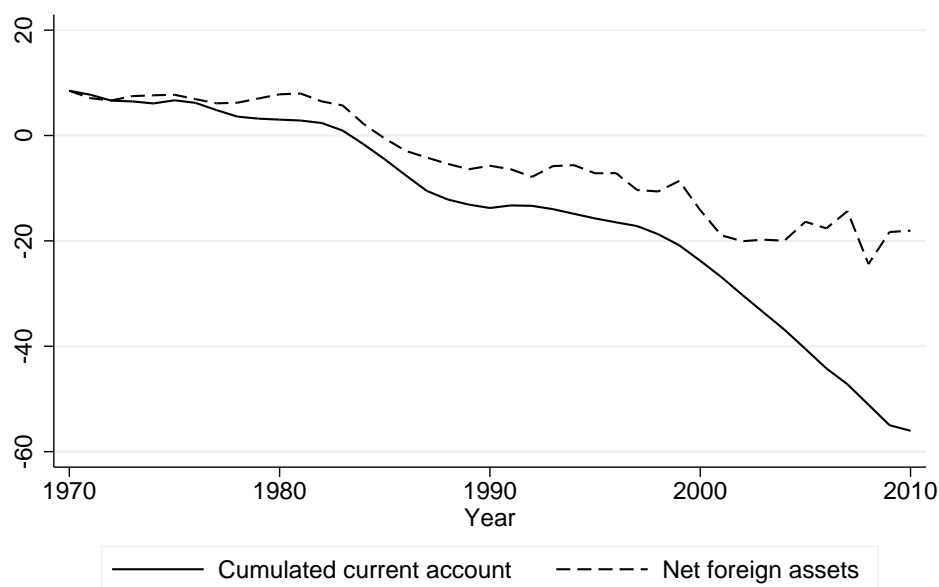
Our two-country model has two key ingredients. Firstly, financial markets are incom-

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<sup>5</sup>While China and Malaysia do not have a fully flexible exchange rate regime, controls on foreign exchange markets have eased over time (see e.g. IMF, 2014) leading to considerable currency appreciations.

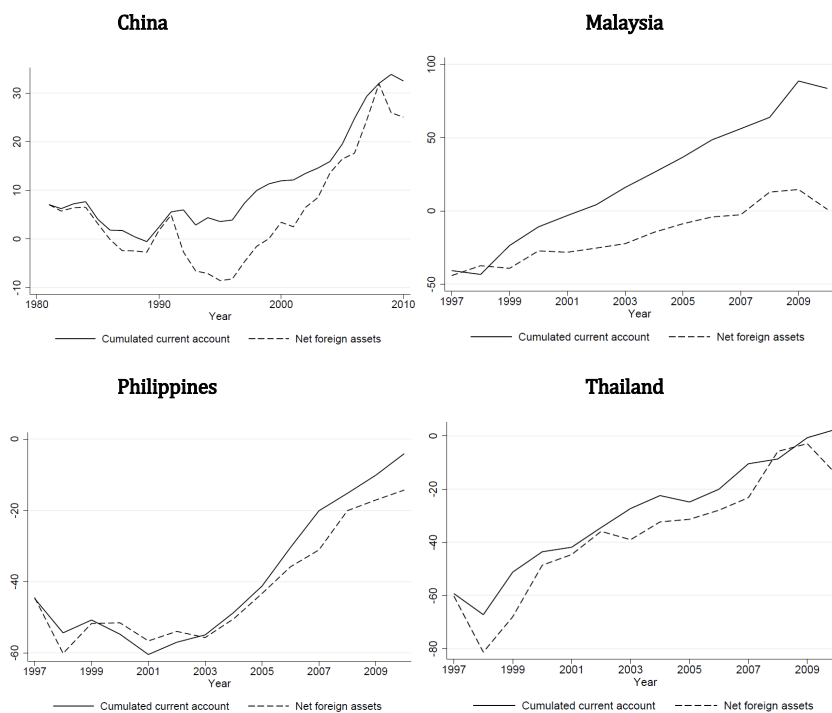
<sup>6</sup>For instance, Tille (2008) analyzes the wealth effects of exchange rate fluctuations assuming that portfolios are exogenous. Benigno (2009) investigates the extent to which the valuation channel due to the exchange rate is desirable from a normative point of view in a model where each country can borrow (lend) in a risk-free nominal bond denominated in its (the other country) currency. As the asset positions and the nominal exchange rate are indeterminate at the steady state, he calibrates the former and restricts the path of the nominal exchange rate.

Figure 1: Net foreign assets' position and cumulated current accounts of the United States as a percentage of GDP, 1970-2010



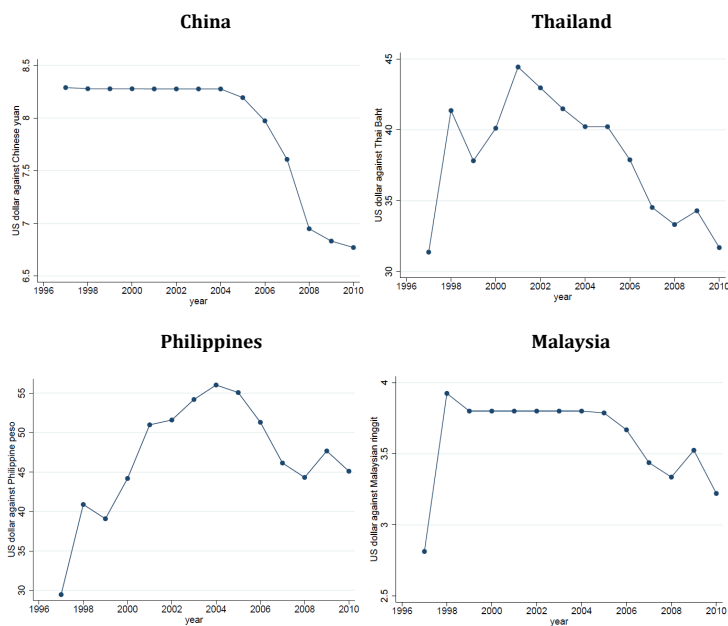
Source: Lane and Milesi-Ferretti's database (2007).

Figure 2: Net foreign assets' position and cumulated current accounts of selected East Asian countries as a percentage of GDP



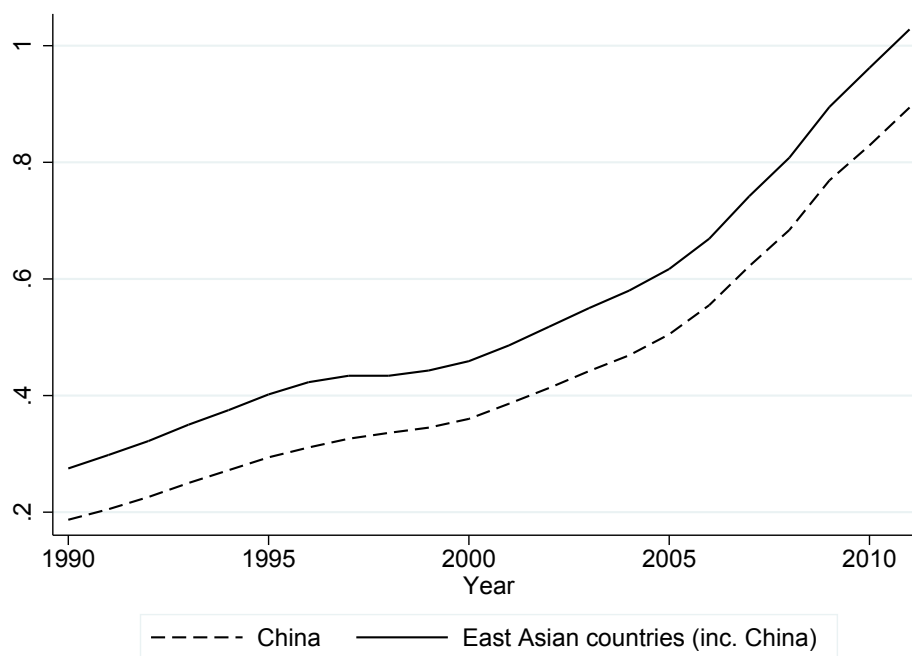
Source: Lane and Milesi-Ferretti database (2007).

Figure 3: The depreciation of the US dollar against the currencies of selected East Asian countries



Source: Lane and Milesi-Ferretti database (2007). Updated to 2011.

Figure 4: The share of world GDP of East Asian countries relatively to the US



Source. IMF World Economic Outlook database. Notes. Gross domestic product based on purchasing-power-parity (PPP) share of world total relatively to the US.

plete. Incomplete markets is an essential feature for open economy models with endogenous portfolio choice: it is well known that complete markets' models are not suitable to explain the dynamics of foreign assets and liabilities (gross positions) as portfolios are constant across states of nature in equilibrium<sup>7</sup>. Our economy is populated by a sequence of overlapping generations, who receive an endowment of a country-specific good when born. Therefore, markets are incomplete in the sense that agents cannot insure against the realization of output that they receive. Secondly, agents lack of a complete set of assets to insure against the risk that they face in the second period of life. The way in which agents can transfer wealth across periods is to buy a portfolio of currencies. We will refer to a country's holdings of the foreign currency as "foreign assets" while the foreign country's holdings of the domestic currency as "foreign liabilities". Because the asset structure is simple, it is very tractable and at the same time it has the necessary elements to capture the currency composition of both the US and emerging countries, for which the majority of foreign assets are denominated in foreign currencies while the majority of foreign liabilities are issued in the domestic currency<sup>8</sup>.

The second ingredient relates to the way in which currencies and goods are traded in the economy. We adopt a cash-in-advance specification as in Lucas (1982), which means that each currency can only buy the country-specific good<sup>9</sup>. However, we require that agents must commit themselves to money holdings before learning the realization of the shock that is going to hit them (in the second period of their lives). In conventional cash-in-advance models, agents purchase money balances *after* the shock is realized and hence the prices of the goods (and, in an open economy, the nominal exchange rate) are known to them. As Lucas (1982) himself suggests, reversing the sequence in which information flows gives rise to a precautionary demand for money as it introduces a further element of uncertainty in agents' decisions. As agents have finite lives in an OLG model, this structure basically implies that only a fraction of agents in the economy (the young) make a decision about portfolios in each period<sup>10</sup>. In this framework, we show that assets'

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<sup>7</sup>See Lucas (1982) for an open economy version of the Lucas asset-pricing model and Judd et al. (2003) for a proof of the same result in a more general version of the model.

<sup>8</sup>Lane et al. (2010) have shown that emerging countries are increasingly similar to developed economies as they are now able to issue the majority of their liabilities in domestic currency, differently from developing economies. Although part of their liabilities are denominated in dollars, East Asian countries (such as China) are net lenders therefore a depreciation of the dollar is still associated with negative valuation effects.

<sup>9</sup>See Alvarez et al. (2009) for a more recent treatment of cash-in-advance models.

<sup>10</sup>As Bacchetta et al. (2010) observe, portfolio decisions are typically not made on a continuous basis but they tend to be infrequent. They show that "infrequent portfolio decisions" can explain the forward discount puzzle in an OLG model.



positions and the nominal exchange rate are determined. If we reversed the way in which information flows in the model, the two currencies would be perfectly substitutable hence we would not be able to pin down a portfolio allocation and the exchange rate. This is nothing but the indeterminacy result pointed out by the seminal paper of Kareken and Wallace (1981)<sup>11</sup>. The presence of cash-in-advance constraints alone is not enough to guarantee the determinacy of the nominal exchange rate, since money has the double role of medium of exchange and store of value in our overlapping-generations setting. In order for the currencies not to be perfect substitutes as stores of value, it is crucial that agents pre-commit to money holdings before the shock is observed<sup>12</sup>.

In this setting, the nominal exchange rate operates through two different, but related channels. Firstly, it has an impact on the portfolio decisions of the agents. In fact, while it is rational for agents to buy currencies that depreciate, as they are relatively cheaper, they also have an incentive to buy those currencies which are expected to have a higher purchasing power in the future. Secondly, fluctuations of the nominal exchange rate have an impact on the net foreign assets position of a country as they generate valuation effects.

This paper provides an interpretation of the stylized facts presented in Figures 1-4, as it explains both the deterioration of the US net external position against emerging market economies and the positive valuation effects that they experienced over the past twenty years as the consequence of emerging countries' increase in their share of world GDP. In fact, we show that the country that runs a current account surplus in equilibrium is the country whose share of world GDP has increased over time. In the numerical exercises, we will focus on China as it is the country whose share of world GDP has increased the most since the early 1990s, because of its rapid output growth. As the current generation in China is wealthier with respect to the previous generation, the young Chinese accumulate more domestic as well as foreign assets than in the past and this causes an improvement of the net foreign assets position of the country, consistently with Figure 2.

The overlapping generations structure is crucial to obtain this result, as the trade balance is driven by the consumption of the old as well as the current generation. In a two-period overlapping generations economy, it is known that the young are net savers whenever their income when old is low as compared to their income when young (i.e.

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<sup>11</sup>Manuelli and Peck (1990) extended Kareken et al.'s result to a stochastic framework. Sargent (1987) showed that the indeterminacy result holds more generally and is not due to the OLG structure. The indeterminacy in Benigno (2009) is also due to the perfect substitutability between the domestic and the foreign bond.

<sup>12</sup>See Appendix A for an illustration of this point.

in Samuelsonian economies). However, at aggregate level, the country could be either borrowing or lending from abroad as part of the aggregate consumption also comes from the old people in the economy. In this sense, it is the comparison between the current and the past distribution of wealth that matters when it comes to determining the net position of the country versus the rest of the world<sup>13</sup>. We also run some panel regressions for a large cross-section of countries for the period 1981-2011 to show that the validity of our mechanism goes beyond explaining the trade balance position of the United versus East Asian economies. In fact, we find that there exists a positive, statistically significant relationship between the current account balance and changes in the share of world GDP. In other words, a country whose share of world GDP has increased with respect to the previous period is more likely to experience a surplus of the current account balance. Excluding the US and China from the sample does not affect this result.

While the United States' net foreign assets position against China deteriorates, the dollar depreciates and this generates positive valuation effects for the US. The mechanism is very intuitive and can be explained as follows. Suppose that the young living both in China and the US expect that the world economy will remain in the current state of nature with high probability. As the Chinese goods are cheaper than in the past due to the country's higher output growth, the demand for the Chinese currency increases because it is expected that it will have a high purchasing power. As the currency appreciates in equilibrium, the value of the foreign currency held by US residents increases relatively to the value of the US currency held abroad. Therefore, the surplus (deficit) country experiences negative (positive) valuation effects, consistently with the stylized facts presented for the US and East Asian economies. The result that valuation effects are stabilizing is obtained under the mild condition that there is some degree of output persistence in the economy. The stabilizing nature of valuation effects is also in line with the empirical findings of Lane et al. (2002) and Devereux et al. (2010).

The numerical results show that valuation effects are quantitatively relevant. While we can explain almost a third of the US-China trade imbalances, valuation effects reduce

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<sup>13</sup>In a framework with infinitely-lived agents, Engel et al. (2006) emphasize the role of the *expected* share of world output as opposed to the *past* share of world GDP. In particular, they show that a country borrows from the rest of the world if the country is expected to grow in the future. Their mechanism is different because the trade balance is entirely driven by the desire of consumption smoothing of the representative agent. While their model is able to capture the US position vis-à-vis other industrialized countries, it falls short of explaining the deterioration of the US external position against East Asian economies since East Asian fast-growing economies are not borrowing from the rest of the world.

the impact of the US current account deficit on the net foreign assets position by more than a half, consistently with the data. This result is not particularly sensitive to our chosen parametrization of the model.

Another contribution of this paper is that the model can generate significantly large expected valuation effects. More precisely, the valuation channel accounts for almost half of the net foreign assets position of the US in our benchmark case. So long as China is expected to grow relatively more than the US, the model suggests that we should expect a deterioration of the US net external position through a trade deficit as well as positive valuation effects through a depreciation of the dollar. Our results are consistent with the empirical literature in pointing out that the valuation channel is crucial for the process of external adjustment (Gourinchas et al., 2007). Our theory also suggests a plausible mechanism driving the adjustment of countries' net foreign assets positions through both the trade balance and the valuation channel: expected growth differentials across countries.

In previous models of asset price-driven valuation effects, the adjustment of the net foreign assets' position entirely operates through the trade balance channel. For instance, this is the case in the analytically tractable model of Pavlova et al. (2007, 2010, 2015), probably because of the logarithmic specification (see also Gourinchas et al., 2015). Tille et al. (2010) and Devereux et al. (2010) resort to higher-order approximations around the deterministic steady state to compute portfolios, as they are indeterminate at the point of approximation. In this class of models, the expected return differential is negligible. Tille et al. (2010) and Ghironi et al. (2015) point out that while expected valuation effects are significant, they are completely offset by movements in dividend income. As a consequence, the trade balance channel is still the only driving force of a country's external adjustment from a quantitative point of view. In this paper, the predictable return differential has instead a large expected component. This is not because there is no stream of interest payments associated with holding money, which could potentially offset exchange rate movements: our result is rather due to our friction in currency markets, which leads to a lack of perfect arbitrage across currencies.

The solution method that we adopt is the numerical computation of the stochastic steady state of the model, which is defined as a time-invariant distribution (across state of nature) of nominal prices, exchange rates, consumption and portfolio allocations. Under some parameter restrictions, we show that the model is partially analytically tractable as

the demand functions have closed-form solutions<sup>14</sup>.

In sections 2, we present the model and define net foreign assets as well as valuation effects in the context of our framework. In section 3, we derive our main analytical result on the relationship between the trade balance and the share of world GDP. In section 4, we parametrize the model to show that it can rationalize the above stylized facts on the dynamics of the net foreign assets of the US and East Asian countries, while in section 5 we conduct a sensitivity analysis to demonstrate that the results are quite robust to alternative parameter specifications. Section 6 concludes the paper.

## 2 The Model

We consider the following two-country overlapping generations economy. In each period, an agent  $h$  with a two-period lifetime is born in each country. Therefore, two young and two old populate the world economy at each date.

The young are born with an endowment of the country-specific good  $\ell$ , which is also the total output of the country. Output is denoted as  $y^\ell(s)$  as it depends on the state of nature realized, where  $s = \{1, \dots, S\}$ . We will use the superscript  $\ell$  to indicate goods and currencies, while we will refer to agents with the subscript  $h$ . We assume that output follows a Markov chain, where transition probabilities are time-invariant and  $\rho(ss')$  indicates the probability of transiting from state  $s$  to  $s'$ . Agents gain utility from the consumption of both goods although they are only endowed with the country-specific good, as in Lucas (1982).

At time 0, the two governments issue fiat money and distribute it to the initial old.  $M^\ell$  is the stock of money issued in country  $\ell$ . As the old have no endowment, money is valued in equilibrium as agents would not be able to consume in their second period of life otherwise. For simplicity, we assume that monetary authorities are inactive after the first period.

Our objective is to characterize the stationary equilibrium of the model, therefore prices will not depend on the history of the shocks but only on the current state of nature. Therefore, agents born in the same state of nature although at different dates have the same consumption allocation.

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<sup>14</sup>Rabitsch et al. (2015) and Coeurdacier et al. (2012) point out that one of the advantages of solving the stochastic steady state of models with endogenous portfolio choice is a higher degree of accuracy of the results, both qualitatively and quantitatively.

Agent  $h$  born in state  $s$  has the following utility function:

$$U_h(s) = \frac{c_{1h}(s)^{1-\gamma}}{1-\gamma} + \beta \sum_{s'} \rho(ss') \frac{c_{2h}(ss')^{1-\gamma}}{1-\gamma} \quad (1)$$

where  $\gamma$  is the coefficient of relative risk aversion and  $c_{1h}(s)$  and  $c_{2h}(ss')$  are the constant elasticity of substitution (CES) aggregators:

$$c_{1h}(s) \equiv \left[ a_h^{1/\sigma} c_{1h}^1(s)^{\frac{\sigma-1}{\sigma}} + a_h^{2/\sigma} c_{1h}^2(s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (2)$$

$$c_{2h}(ss') \equiv \left[ a_h^{1/\sigma} c_{2h}^1(ss')^{\frac{\sigma-1}{\sigma}} + a_h^{2/\sigma} c_{2h}^2(ss')^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (3)$$

where  $a_h^1 + a_h^2 = 1$ ,  $\sigma > 0$  and  $\sigma \neq 1$ .

Taking as given the vector of transition probabilities, the goods' prices and the nominal exchange rate, agent  $h$  born in state  $s$  then chooses the consumption vectors and the portfolio of currencies that maximize (1) subject to (2), (3) and the following constraints:

$$\bar{m}_h^1(s) + e(s)\bar{m}_h^2(s) = w_h(s) \quad (4)$$

$$m_h^1(s) + p^1(s)c_{1h}^1(s) = \bar{m}_h^1(s) \quad (5)$$

$$m_h^2(s) + p^2(s)c_{1h}^2(s) = \bar{m}_h^2(s) \quad (6)$$

$$p^1(s')c_{2h}^1(ss') = m_h^1(s) \quad \forall s' \quad (7)$$

$$p^2(s')c_{2h}^2(ss') = m_h^2(s) \quad \forall s' \quad (8)$$

Agent  $h$  is born with an initial wealth  $w_h(s)$ , which is equal to the value of the domestic output expressed in units of currency 1 (the numéraire):  $w_1(s) \equiv p^1(s)y^1(s)$  and  $w_2(s) \equiv p^2(s)e(s)y^2(s)$ .  $p^1(s)$  ( $p^2(s)$ ) is the price of good 1 (2) expressed in units of currency 1 (2). We assume that the law of one price holds, hence producers set prices in their own currency<sup>15</sup>.  $e(s)$  is the price of currency 2 in units of currency 1 or the nominal exchange rate. Therefore, we say that if  $e(s)$  rises then currency 2 (1) appreciates (depreciates). With his wealth, the agent buys the two currencies for the purpose of financing the consumption of the domestic and foreign good as well as for saving purposes.  $\bar{m}_h(s)$  is the portfolio of currencies held at the beginning of the period. The second and the third constraints are the cash-in-advance constraints faced by the young, where  $m_h(s)$  is the end-of-period portfolio.

For the old agents, the timing is structured as follows. Firstly, the state of nature realizes. Secondly, the previously accumulated currencies are spent in the respective goods'

<sup>15</sup>This is not a restrictive assumption, considering that this is a flexible price, long-run model. For instance, Campa et al. (2006) have shown that there is full exchange rate pass-through in the long-run for many types of imported goods.

markets. As a consequence of the fact that they cannot adjust their previous portfolio decision, the old effectively face as many constraints as the number of goods for each state of nature (instead of a single budget constraint). As we explained in the introduction, the way in which information flows in the model introduces a further element of risk. As agents pre-commit to money holdings *before* the value of the shock is known to them, agents pick a portfolio of currencies forming expectations about the purchasing power of the two currencies next period. This structure ensures the determinacy of portfolios and the nominal exchange rate in a framework where currencies are used not only as a media of exchange, but also as stores of value<sup>16</sup>.

We can consolidate the constraints of the young (4), (5) and (6) into a single budget constraint and rewrite the problem as follows:

$$p^1(s)c_{1h}^1(s) + p^2(s)e(s)c_{1h}^2(s) - w_h(s) = -m_h^1(s) - e(s)m_h^2(s) \quad (9)$$

$$p^1(s')c_{2h}^1(ss') = m_h^1(s) \quad \forall s' \quad (10)$$

$$p^2(s')c_{2h}^2(ss') = m_h^2(s) \quad \forall s' \quad (11)$$

For analytical convenience, we will assume that the intertemporal elasticity of substitution, which is the inverse of the coefficient of relative risk aversion  $\gamma$ , is equal to the elasticity of substitution between the traded goods  $\sigma$ <sup>17</sup>:

**Assumption 1**  $\frac{1}{\gamma} = \sigma$ .

Let  $\lambda_h(s)$  be the multiplier associated to the young's budget constraint,  $\lambda_h^\ell(ss')$  the multiplier of the constraint of the old related to good  $\ell$  in state  $s'$ . The necessary and sufficient

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<sup>16</sup>See Appendix A for further considerations.

<sup>17</sup> This parameter restriction allows us to derive the demand functions for the two currencies analytically, which is very helpful to gain intuition of the main mechanisms. In the sensitivity analysis, we will show that the qualitative and quantitative results obtained under this restriction are robust for commonly assumed values of  $\gamma$  and  $\sigma$ .

conditions for a maximum are the following:

$$c_{1h}^1(s) : a_h^{1\frac{1}{\sigma}} c_{1h}^1(s)^{-\frac{1}{\sigma}} = \lambda_h(s) p^1(s) \quad (12)$$

$$c_{1h}^2(s) : a_h^{2\frac{1}{\sigma}} c_{1h}^2(s)^{-\frac{1}{\sigma}} = \lambda_h(s) p^2(s) e(s) \quad (13)$$

$$c_{2h}^\ell(ss') : \beta a_h^{\ell\frac{1}{\sigma}} \rho(ss') c_{2h}^\ell(ss')^{-\frac{1}{\sigma}} = \lambda_h^\ell(ss') p^\ell(s') \quad \forall \ell, s' \quad (14)$$

$$m_h^1(s) : -\lambda_h(s) + \sum_{s'} \lambda_h^1(ss') = 0 \quad (15)$$

$$m_h^2(s) : -\lambda_h(s) e(s) + \sum_{s'} \lambda_h^2(ss') = 0 \quad (16)$$

$$\begin{aligned} \lambda_h(s) : & p^1(s) c_{1h}^1(s) + p^2(s) e(s) c_{1h}^2(s) - w_h(s) + \\ & + m_h^1(s) + e(s) m_h^2(s) = 0 \end{aligned} \quad (17)$$

$$\lambda_h^1(ss') : p^1(s') c_{2h}^1(ss') - m_h^1(s) = 0 \quad \forall s' \quad (18)$$

$$\lambda_h^2(ss') : p^2(s') c_{2h}^2(ss') - m_h^2(s) = 0 \quad \forall s' \quad (19)$$

In Appendix A, we show how to find the following closed-form solutions for the agents' demand functions for the two currencies:

$$m_h^1(s) = \frac{a_h^1 \beta^\sigma \left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{A_h(s)} w_h(s) \quad (20)$$

$$m_h^2(s) = \frac{a_h^2 \beta^\sigma e(s)^{1-\sigma} \left[ \sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{A_h(s)} \frac{w_h(s)}{e(s)} \quad (21)$$

where

$$\begin{aligned} A_h(s) \equiv & a_h^1 p^1(s)^{1-\sigma} + a_h^2 [p^2(s) e(s)]^{1-\sigma} + a_h^1 \beta^\sigma \left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma + \\ & + a_h^2 \beta^\sigma e(s)^{1-\sigma} \left[ \sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma \end{aligned}$$

Agent  $h$ 's demand functions for the goods can be derived using (20), (21) and the budget constraints<sup>18</sup>:

$$c_{1h}^1(s) = \frac{a_h^1 p^1(s)^{-\sigma}}{A_h(s)} w_h(s) \quad \forall \ell \quad (22)$$

$$c_{1h}^2(s) = \frac{a_h^2 [p^2(s) e(s)]^{-\sigma}}{A_h(s)} w_h(s) \quad \forall \ell \quad (23)$$

$$c_{2h}^1(ss') = \frac{a_h^1 \beta^\sigma \left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{A_h(s)} \frac{w_h(s)}{p^1(s')} \quad \forall s' \quad (24)$$

$$c_{2h}^2(ss') = \frac{a_h^2 \beta^\sigma e(s)^{-\sigma} \left[ \sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{A_h(s)} \frac{w_h(s)}{p^2(s')} \quad \forall s' \quad (25)$$

<sup>18</sup>The procedure to calculate the demand functions when young is provided in Appendix A.

As preferences are homothetic, the demand for each good is a linear function of wealth where the multiplicative term is a complicated non-linear function of current and future prices as well as the nominal exchange rate.

## 2.1 The role of the nominal exchange rate: partial equilibrium

Using equations (14), (15) and (16), we can obtain the following expression for the nominal exchange rate:

$$e(s) = \frac{a_h^2 \sum_{s'} \rho(ss') \frac{c_{2h}^2(ss')^{-\frac{1}{\sigma}}}{p^2(s')}}{a_h^1 \sum_{s'} \rho(ss') \frac{c_{2h}^1(ss')^{-\frac{1}{\sigma}}}{p^1(s')}} \quad s = 1, \dots, S \quad (26)$$

In our model, the nominal exchange rate is a forward-looking variable, as it depends on the expected marginal utilities derived from the consumption of the two goods as well as from the expected purchasing power of the two currencies. In fact,  $\frac{1}{p^\ell(s')}$  gives how many units of good  $\ell$  we can afford in state  $s'$  per unit of currency  $\ell$  held. In other words, the nominal exchange rate is the ratio of the expected purchasing power of currency 2 over the expected purchasing power of currency 1, weighted by agent  $h$ 's marginal utilities. The more a currency can buy tomorrow relatively to the other currency, the higher will be its price today. This means that the nominal exchange rate follows some sort of asset pricing equation, given that the currencies are used to transfer wealth across periods<sup>19</sup>.

Let us now look at the portfolio decision of an agent in more detail. We combine the demand for the two currencies (20) and (21) to get:

$$\frac{m_h^1(s)}{m_h^2(s)} = \underbrace{e(s)^\sigma}_{\text{nominal exchange rate}} \cdot \underbrace{\frac{a_h^1}{a_h^2}}_{\text{goods' weights}} \cdot \underbrace{\frac{\left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{\left[ \sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}}_{\text{expected nominal prices}} \quad (27)$$

The portfolio decision of each agent depends on three sets of variables: the nominal exchange rate, the weights of the two goods in the utility function, the expected future prices of the two goods.

Firstly, as currency 2 becomes more expensive ( $e(s)$  increases) the relative demand for currency 1 increases. Secondly, the relative demand for currency 1 rises with the weight of good 1 in agent  $h$ 's utility function. The equation also shows that the impact

<sup>19</sup>In the cash-in-advance literature, the spot exchange rate simply depends on the *current* realization of the stochastic variables and not on expectations of future variables (see e.g. Lucas (1982)). This is due to the transaction role that it is attributed to money, which is only used to carry out exchange in a given period. In Lucas (1982), money is a “veil” and the exchange rate does not ultimately affect the real allocation, which is the same as in the barter economy.



of future prices on portfolio choices depends on the degree of substitutability between the two goods, which in turn determines the degree of substitutability between the two currencies. In general, if it is expected that a good will be relatively cheaper, then the associated currency will have a higher purchasing power. If  $\sigma > 1$ , the demand for such currency would increase as the substitution effect is sufficiently high.

Obviously, these arguments about the role of the nominal exchange rate on the portfolio choice of the agents are of a partial equilibrium nature, as agents take prices and the nominal exchange rate as given. Below, we will show the importance of general equilibrium analysis as the nominal exchange rate moves in equilibrium to offset the expected price differentials across countries.

## 2.2 Stationary equilibrium

**Definition 1** *A stationary equilibrium is a system of prices and nominal exchange rates  $(p, e) \in \mathbb{R}_{++}^{2S} \times \mathbb{R}_{++}^S$ , consumption allocations and portfolios  $(c_{1h}(s), c_{2h}(ss'), m_h(s)) \in \mathbb{R}_{++}^2 \times \mathbb{R}_{++}^{2S} \times \mathbb{R}_{++}^2$  for every  $h = 1, \dots, H$  and  $s = 1, \dots, S$  such that:*

- (i) *agent  $h$  maximizes his utility function (1) subject to the budget constraints (9), (10) and (11) in every  $s$ ;*
- (ii)  $c_1^\ell(s) + c_2^\ell(s's) = y^\ell(s) \quad \forall s, s' \text{ and } \forall \ell$
- (iii)  $\sum_h m_h^\ell(s) = M^\ell \quad \forall s, \ell$

where  $c_1^\ell(s) \equiv \sum_h c_{1h}^\ell(s)$  and  $c_2^\ell(s's) \equiv \sum_h c_{2h}^\ell(s's)$ .

In the previous section, we have shown how to compute analytically the demand functions for the goods and the currencies. Therefore, the number of endogenous variables that we need to compute reduces to  $3S$ , i.e. the two nominal prices and the exchange rate in each state of nature. The number of equations is instead  $2S + 2S^2$ .  $2S$  refer to the two money markets, which have to clear in each state of nature while  $2S^2$  are the two goods' markets, which have to clear for any pair of  $s$  and  $s'$  as the consumption of the old depends on the state when born as well as the current state. In Appendix B, we show that the goods' markets equations that are apparently in excess are actually redundant, so that solving the model actually reduces to handling a non-linear system of  $3S$  equations and unknowns.

Before solving the model, we introduce some key definitions and make some useful remarks.

### 2.3 Portfolio rebalancing and trade imbalances: a unified view

To start with, let us define the balance of trade of country 1 in state  $s$ , where  $s'$  is the state of nature in the previous period<sup>20</sup>:

$$TB_1(s's) \equiv p^1(s)[y^1(s) - c_{11}^1(s) - c_{21}^1(s's)] - p^2(s)e(s)[c_{11}^2(s) + c_{21}^2(s's)]$$

Notice that the sign of the balance of trade depends on the choices that the young make in the current period, but also on the choices made by the current old in the previous period. Substituting the budget constraints into the trade balance equation, it should be immediate that the above definition can be rewritten as:

$$TB_1(s's) = m_1^1(s) - m_1^1(s') + e(s)[m_1^2(s) - m_1^2(s')] \quad (28)$$

This leads us to the following remarks:

**Remark 1** *The balance of trade is zero if: (i) portfolios are constant across states of nature; (ii) this period's realized state is the same as last period's.*

Equation (28) shows that there is a close relationship between currency markets and the goods' markets. If, for some reason, there is no portfolio rebalancing in equilibrium, then the balance of trade is always balanced. In Appendix B, we show that this behaviour occurs e.g. when utility functions are logarithmic. The demand functions become extremely simple as they do not depend on future prices, and the model is fully tractable. Constant portfolios imply that the consumption of an old person does not depend on the state in which he is born but only on the state realized when old. As we explained in the introduction, this is a prediction at odds with the reality of international financial markets. When the elasticity of substitution is different than one, we will show that agents born in different states of nature have different demands for the goods which is then reflected in their demand for the two currencies<sup>21</sup>.

The second part of the remark is related to the result of Polemarchakis et al. (2002) for deterministic OLG economies. In a monetary union with no uncertainty, they showed

<sup>20</sup>Obviously, by Walras Law we have that  $TB_2(s's) = -TB_1(s's)$ .

<sup>21</sup>Our finding for the log case is related to Cass and Pavlova (2004), who showed that the matrix of portfolio returns is degenerate in a two-period economy with  $N$  Lucas trees. A similar result holds in the infinite-horizon setting of Pavlova and Rigobon (2007, 2010), who then introduce demand shocks to generate time-varying portfolios.

that the balance of trade is always in equilibrium at the monetary steady state. In this paper, the monetary steady state is stochastic and trade imbalances are possible whenever  $s \neq s'$ .

Next, we decompose the trade balance equation to highlight valuation effects and the change in net foreign assets.

## 2.4 Net foreign assets dynamics and a decomposition of valuation effects

In this section, we explore the relationship between net foreign assets, the balance of trade and valuation effects. Consider the balance of trade of country 1 in state  $s's$ , where  $s'$  is the past state and  $s$  is the current state, as defined in the previous section (equation (28)). Using the fact that  $m_1^1(s) + m_2^1(s) = M^1$  for every  $s$ , we can rewrite it as follows:

$$TB_1(s's) = \underbrace{m_2^1(s')}_{FL_1(s')} - \underbrace{m_2^1(s)}_{FL_1(s)} + \underbrace{e(s)m_1^2(s)}_{FA_1(s)} - \underbrace{e(s)m_1^2(s')}_{\text{current value } FA_1(s')} \quad (29)$$

where  $FA(s)$  and  $FL(s)$  stand respectively for “foreign assets” and “foreign liabilities”, which, in this context, are a country’s holdings of foreign currency and the foreign country’s holdings of the domestic currency. Next, define net foreign assets as  $NFA(s) \equiv FA(s) - FL(s)$  and rewrite the above as follows:

$$NFA_1(s) = \text{current value } NFA_1(s') + TB_1(s's) \quad (30)$$

Equation (30) states that the end-of-period net foreign assets in country 1 is equal to the current value of the net foreign assets accumulated in the previous period plus the trade balance<sup>22</sup>.

The next step is to rewrite equation (29) in order to highlight the valuation effects in this model. We can do that by summing and subtracting the foreign assets of country 1 in the previous state ( $e(s')m_1^2(s')$ ) in the right hand side and using the definition of net foreign assets:

$$TB_1(s's) = NFA_1(s) - NFA_1(s') + [e(s') - e(s)]m_1^2(s') \quad (31)$$

This equation can be rewritten as:

$$\Delta NFA_1(s's) = TB_1(s's) + r(s's)FA_1(s') = TB_1(s's) + VAL_1(s's) \quad (32)$$

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<sup>22</sup>This equation is equivalent to equation (1) in Gourinchas and Rey (2007, footnote 2).

where

$$r(s's) \equiv R(s's) - 1 \equiv \frac{e(s)}{e(s')} - 1 \quad (33)$$

$$VAL_1(s's) \equiv r(s's)FA_1(s') \quad (34)$$

The change in the net foreign assets position of country 1 will be determined by the behaviour of the balance of trade and the valuation effects, where  $r(s's)$  is the return on the foreign assets accumulated in the previous period<sup>23</sup>. In this model, valuation effects are entirely determined by exchange rate movements. If foreign currencies have appreciated with respect to the past (i.e.  $e(s) > e(s')$ ), then the return on the foreign assets accumulated in the previous period is positive and therefore we say that the country experiences positive valuation effects<sup>24</sup>. Conversely, a country experiences negative valuation effects if the foreign currency has depreciated.

Gourinchas et al. (2015) recently stressed that one of the challenges for open economy models with endogenous portfolio choice is to be able to come up with a model that generates substantial *expected* valuation effects as opposed to *unexpected* valuation effects. In fact, Gourinchas et al. (2007) have shown empirically that the valuation channel is a critical component of the process of external adjustment of the US, and the exchange rate component is a fundamental force behind that.

To be able explore this issue later on, we decompose  $VAL_1(s's)$ , which are the valuation effects that country 1 would actually experience in the transition from state  $s'$  to state  $s$ , into an expected and an unexpected component. Firstly, we can write the valuation effects expected in state  $s'$  as follows:

$$E_{s's}VAL_1(s's) \equiv FA_1(s')E_{s's}r(s's) \quad (35)$$

where  $E_{s's}$  is the expectation operator which is conditional on the world economy being in state  $s'$ . Equation (35) indicates that expected valuation effects are not zero in the model as long as the expected return on the foreign assets' position, which is driven by nominal exchange rates fluctuations, is different from zero.

Hence, the unexpected component is simply the difference between the actual (realized)

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<sup>23</sup>Notice that there is no net income from abroad and therefore the trade balance position is equivalent to the current account position.

<sup>24</sup>As the price of the foreign asset is defined in units of the domestic asset, i.e. the exchange rate, the above rate of return has to be interpreted as the return of foreign assets relatively to the return on foreign liabilities.

and the expected valuation effects:

$$\begin{aligned}
\underbrace{UNVAL_1(s's)}_{\text{unexpected val. effects}} &= \underbrace{VAL_1(s's)}_{\text{actual val. effects}} - \underbrace{E_{s's}VAL_1(s's)}_{\text{expected val. effects}} = \\
&= FA_1(s')[r(s's) - E_{s's}r(s's)]
\end{aligned} \tag{36}$$

We can also derive a forward-looking expression for the net foreign assets position of country 1 in state  $s'$  to highlight the role of expected valuation effects in the dynamics of the net foreign assets' position of the country. The change in the net foreign assets position between  $s$  and  $s'$  must satisfy the following equation:

$$NFA_1(s) - NFA_1(s') = TB_1(s's) + VAL_1(s's) \quad \forall s$$

Multiplying each equation by the probability that state  $s$  realizes next period given that the current state is  $s'$  and summing across the  $S$  equations, we get:

$$NFA_1(s') = E_{s's}NFA_1(s) - E_{s's}TB_1(s's) - E_{s's}VAL_1(s's) \tag{37}$$

Equation (37) shows that the net foreign assets position of country 1 in state  $s'$  can be written as a function of the expected net foreign assets position, the trade balance and valuation effects<sup>25</sup>.

Before showing the behavior as well as the quantitative importance of expected valuation effects in the dynamics of net foreign assets, in section 3 we show that portfolio rebalancing, and therefore trade imbalances, occurs in equilibrium whenever there are changes in countries' share of world GDP.

### 3 The distribution of world GDP, portfolio rebalancing and trade imbalances

In this section only, we assume that there is no home bias ( $a_h^1 = a_h^2$ ) as this case is very helpful to develop some key intuitions about the model. The following Proposition establishes that there is a strong relationship between the distribution of world GDP across countries, portfolio holdings and trade imbalances when preferences are identical across countries.

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<sup>25</sup>In Gourinchas et al. (2007), the first term in the right-hand side does not appear as their analytical expression is based on the assumption that the no-Ponzi game condition holds. While this is always the case when agents are infinitely-lived, we do not have to impose such restriction on the economy in an overlapping-generations setting. We will see that this will have important consequences for the dynamics of net foreign assets.

**Proposition 1** Assume that  $a_h^1 = a_h^2$ . (i) A country's portfolio holdings at the end of a period are linearly related to its current share of world GDP.

(ii) If a country has a higher (lower) share of world GDP with respect to the past, it runs a trade surplus (deficit).

**Proof.**

- (i) The demand of agent  $h$  for the two currencies is linear in wealth (see equations (20) and (21)):

$$m_h^\ell(s) = k^\ell(s)w_h(s)$$

where  $k^\ell(s)$  is identical across agents if  $a_h^1 = a_h^2$ . Summing across  $h$  and assuming that the money markets clear, we get the following equation:

$$M^\ell = k^\ell(s) \sum_h w_h(s)$$

Dividing the first equation by the second equation and rearranging, we obtain the desired result:

$$m_h^\ell(s) = \frac{w_h(s)}{w(s)} M^\ell \quad \ell = 1, 2$$

where  $w(s) = \sum_h w_h(s)$ <sup>26</sup>.

- (ii) Suppose that today's realized state is  $s$  and yesterday's state was  $s'$  and assume that  $\frac{w_h(s)}{w(s)} > \frac{w_h(s')}{w(s')}$ . Our previous result implies that:

$$m_h^\ell(s) > m_h^\ell(s') \quad \ell = 1, 2$$

As the young born in the current period hold a higher share of both currencies than the previous generation, then country  $h$  runs a trade surplus by equation (28). The other case can be worked out in a similar way.

■

The end-of-the-period wealth of the young is equal to their total money holdings, as the two currencies are the means by which they save and therefore finance future consumption. Therefore, Proposition 1 suggests that the distribution of world wealth at the end of a period is strongly related to the distribution of world GDP<sup>27</sup>. If the distribution of world

<sup>26</sup>World GDP is defined as the sum of countries' nominal GDP expressed in units of the numéraire currency.

<sup>27</sup>Because nominal interest rates are zero, then domestic GDP is equal to domestic income. Therefore, the distribution of world GDP is also equal to the distribution of world income.

GDP changes across states of nature, then portfolio rebalancing occurs over time and the distribution of wealth will change too.

Proposition 1 sheds further light on the behavior of the trade balance. If a country is in surplus, it is because the young are relatively wealthier with respect to the past and hence accumulate more assets, although this does not rule out the possibility that a country is poorer than the other country in all states of nature<sup>28</sup>. Therefore, our model offers a novel explanation of the fact that emerging countries run trade surpluses against the United States: global imbalances simply reflect the rise of emerging countries in the world economy, as Figure 4 suggests.

Since the share of world GDP is calculated in nominal terms, a change in a country's share of world GDP can be attributed partly to changes in output and partly to changes in prices. Our numerical results will show that output changes dominate the terms of trade effect as long as the elasticity of substitution between traded goods is greater than 1, which is supported by empirical evidence. Therefore, the reason behind the Chinese surplus is that China's real GDP has grown more than in the US. This has implied that the distribution of world GDP has changed in favour of China over the past 20 years, as Figure 4 suggests. The model shows that this is at the heart of the US-China imbalances.

Table 1 shows that the usefulness of our mechanism is not limited to capturing the dynamics of the trade balance between the United States and East Asian countries.

We test the hypothesis suggested by Proposition 1, i.e. that the trade balance is positively correlated with changes in the share of world GDP, estimating the following fixed-effects model:

$$\text{current account balance}_{i,t} = \alpha_i + \beta_1 \cdot \Delta \text{share of the world GDP}_{i,t} + \beta_2 \cdot \text{country size}_{i,t} + \varepsilon_{i,t}$$

where

$$\Delta \text{share of the world GDP}_{i,t} = \text{share of the world GDP}_{i,t} - \text{share of the world GDP}_{i,t-1}$$

We use the current share of the world GDP as a proxy for country size. Our results show that this relationship holds and is statistically significant for a large cross-section of countries. In columns (3) and (4), we perform a couple of robustness checks. Firstly, we remove developing countries from the sample. The reason is that there is a large

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<sup>28</sup>On the other hand, the poor country is always in trade deficit in cash-in-advance models under isoelastic utility (see Eugeni (2013) for a derivation). The reason is that the sign of the trade balance only depend on the current shock, and not on the past.

Table 1: Panel regression of the current account balance, 1981-2011.

	(1)	(2)	(3) Ex. developing	(4) Ex. China and US
$\Delta$ share of world GDP	101.0027*** (6.6998)	57.3803*** (6.9661)	58.937*** (11.3750)	10.0902*** (4.6484)
Country size		22.8006*** (1.30894)	22.8603*** (2.1037)	-20.1991*** (0.9534)
Constant	0.16445 (0.3322)	-13.5959*** (0.85319)	-32.7898*** (3.1572)	10.3064*** (0.4612)
$R^2$	0.0956	0.1045	0.1185	0.0752
No. countries	183	183	69	181
No. observations	5,038	5,038	1,951	4,992

*Source.* IMF World Economic Outlook database. The main dependent variable is calculated using “Gross domestic product based on purchasing-power-parity (PPP) share of world total”. Data are until 2011 as shares of world GDP are estimates after that date for many countries. *Notes.* The standard errors are reported in parenthesis. \*\*\* denotes significance at the 1% level, \*\* at the 5% level and \* at the 10% level. All regressions are estimated with fixed effects.

cluster of observations for which the change in the share of world GDP is zero, and this is typically the case for many developing countries. It can be observed that the regression results are not significantly affected. Secondly, we remove instead the United States and China since these two countries run the largest trade imbalances in the sample. While the estimated coefficient is significantly smaller, the qualitative prediction as well as the statistical significance are not challenged.

Finally, we conclude this section by commenting how agents allocate savings across the two currencies in the model. According to Proposition 1, agents do not have very sophisticated portfolio strategies as they hold the same share of both money stocks. One reason is that we have assumed away “home bias” in the preferences, therefore agents hold the two currencies in the same share as they like the two goods equally. We relax this assumption in the numerical exercises below. The second reason is the “shock absorbing” role that the nominal exchange rate plays in this model.



Plugging the demand for the two currencies into the equilibrium conditions for the money market, we get the following two equations:

$$M^1 = \frac{1}{2} \frac{\beta^\sigma \left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{A(s)} \sum_h w_h(s) \quad (38)$$

$$e(s) M^2 = \frac{1}{2} \frac{\beta^\sigma e(s)^{1-\sigma} \left[ \sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{A(s)} \sum_h w_h(s) \quad (39)$$

Combining equations (38) and (39), we obtain the following expression for the exchange rate:

$$e(s) = \left( \frac{M^1}{M^2} \right)^{\frac{1}{\sigma}} \frac{\sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}}}{\sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}}} \quad s = 1, \dots, S \quad (40)$$

Although the above expression is not a closed-form solution, we can gain some intuition about the role of the nominal exchange rate and the importance of general equilibrium analysis as opposed to partial equilibrium analysis. Recall the equation that linked the portfolio choice of the agents to the nominal exchange rate (equation (27)). In a partial equilibrium setting, an expected increase in the price of good 1 means that the purchasing power of currency 1 will be lower in the future, therefore the relative demand for currency 1 falls (provided that the elasticity of substitution is bigger than 1). In equilibrium, the nominal exchange rate will behave in such a way to counteract expectations on price movements. In fact, equation (40) shows that currency 1 would depreciate to compensate the effect on the demand for currency 1 of its fall of purchasing power. As a consequence, agents choose the currencies in equal shares in equilibrium in the absence of home bias (Proposition 1). In the context of the numerical exercises, we will explain in more detail how output shocks generates nominal exchange rate fluctuations through fluctuations of nominal prices.

## 4 The deterioration of the United States' external position against China: the role of valuation effects

The aim of this section is to show that our two-country model is able to explain the dynamics of the net external position of the United States and China over the past twenty years as the consequence of China's increase in the share of world GDP, which is due to China's higher GDP growth. The intention is not to provide a fully-fledged calibration exercise, as the model is stylized in many aspects. The purpose is rather to

demonstrate that the model can offer an interpretation of the stylized facts presented in the introduction, as well as a clean mechanism that indicates why and how exchange rate-driven valuation effects contribute to the dynamics of the net external position of a country.

The reasons why we choose the United States and China for our numerical exercise are several. Firstly, China is widely known as one of the main creditors of the US and the US deficit against China account for a significant fraction of the overall US current account deficit (e.g. Eugeni, 2015). Second, the US has lost a considerable share of world GDP in favour China (Figure 4). Moreover, the US-China imbalances are persistent and our two-period OLG model is especially suitable to capture low-frequency trends in international financial markets. The currency composition of the US and China's balance sheet can also be captured by our model, as their foreign assets are mainly denominated in foreign currencies while foreign liabilities are mainly denominated in the domestic currency. According to the Benetrix et al. (2015) database, 64% of US foreign assets were denominated in foreign currencies while 88% of US foreign liabilities were denominated in dollars in 2010<sup>29</sup>. As far as China is concerned, 100% of the Chinese foreign assets are denominated in foreign currency, 61% of which are dollar denominated. On the other hand, 76% of Chinese liabilities were issued in renminbi in 2010. This reflects a general trend which sees emerging market economies increasingly able to borrow in their domestic currency (Lane and Shambaugh, 2010)<sup>30</sup>.

Since agents live for two periods in our OLG economy, we assume that a period is 20-years long. As we wish to explain the deterioration of the US external position against China over the past 20 years, we adopt the following strategy. We consider an economy with two states of nature, where state 1 corresponds to the state of the world economy in 1990 while state 2 is the state of the world economy in 2010. Therefore, we will focus on what happens in the world economy in the transition from state 1 to state 2<sup>31</sup>.

We report the parameter values that we choose for the numerical exercise in Table 2.

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<sup>29</sup>The first figure reflects the fact that many developing economies still borrow in US dollars as they are unable to issue debt in domestic currency-denominated assets.

<sup>30</sup>Another signal of the increased ability of emerging countries to borrow in their own currency is that a third of the foreign currency-denominated US foreign assets are denominated in currencies other than the Euro, the Yen, the Pound and the Swiss Franc. Therefore, these are assets held in emerging economies and denominated in local currencies.

<sup>31</sup>This is not to argue that the world economy can only be in a state that matches the situation of the world economy either of the 1990 or the 2010. However, a two-states example is enough to illustrate our arguments while adding more states of nature would not provide neither more information nor intuition.

We take the real GDP per capita of the United States and China in 1990 and 2010 to parametrize output in the two states<sup>32</sup>. Notice that while US output has grown by 32% over the 20-years period, China has grown by 384%. Although China has experienced higher growth over time, real GDP per capita is still much lower than the US.

We normalize both money supplies to 1 since the level of the money supply does not affect the real allocation<sup>33</sup>.

Since the model is not aimed at explaining high-frequency data, our value for the elasticity of substitution between traded goods is more in line with the empirical work based on low-frequency data, which found values in the range between 4 and 15 (see Ruhl, 2008). In the next section, we will show that our results are robust to different parameter values for the elasticity of substitution as long as  $\sigma$  is greater than 1. Notice that this rules out episodes of “immiserizing growth”. In fact, when  $0 < \sigma < 1$ , a country that experiences a positive output shock (everything else equal) is poorer in nominal terms since the terms of trade effect dominates changes in output<sup>34</sup>.

The discount factor is set equal to 1 and identical across countries (we consider a more standard value in the sensitivity analysis). We assume that the probability that output tomorrow is the same as today is 0.9, which implies that output is somewhat persistent.  $1 - \rho(ss)$  can also be interpreted as the probability that China catches up with the US

<sup>32</sup>We take output-side real GDP at chained PPPs and population data from the Penn World Tables 8.0.

<sup>33</sup>It can be checked that e.g. changing the money supply of country 1 by a fraction  $\lambda$  will increase  $p_1$  and  $e$  by exactly the same fraction with no effects on consumption allocations and the various components of the balance of payments once normalized as percentages of GDP.

<sup>34</sup>See e.g. Cole and Obstfeld (1991).

Table 2: Parameter Values. Baseline model.

US real GDP in 1990	$y^1(1) = 31,432$
US real GDP in 2010	$y^1(2) = 41,627$
Chinese real GDP in 1990	$y^2(1) = 2,005$
Chinese real GDP in 2010	$y^2(2) = 7,693$
Money supply	$M_1 = M_2 = 1$
Elasticity of substitution	$\sigma_1 = \sigma_2 = 4$
Share of home goods	$a_1^1 = a_2^2 = 0.72$
Discount factor	$\beta_1 = \beta_2 = 1$
Output persistence	$\rho(ss) = 0.9$

as of 1990<sup>35</sup>. Although assuming that the persistence parameter is 0.9 is standard in the real business cycle literature, this might be questionable in a long-run model such as this one. Hence, we will conduct various robustness exercises on this parameter. Finally, we set the share of home goods in consumption as equal to 0.72 as in Corsetti et al. (2008).

In the next section, we will also study a more general version where the elasticity of intratemporal substitution (between traded goods) is allowed to differ from the elasticity of intertemporal substitution: while the model loses analytical tractability, the qualitative as well as the quantitative results are not significantly affected.

#### 4.1 The stabilizing role of valuation effects

We explain the solution method and report the equilibrium prices, as well as a calculation of the terms of trade, in Appendix B.

Country 1 (the US) experiences an improvement of the terms of trade in the transition from state 1 to state 2, as the price of imports falls relatively to the price of exports. This is due to a supply effect, as output in country 2 (China) has increased relatively more than output in country 1. At the same time, currency 1 depreciates (see the Appendix).

The main intuition behind the depreciation of the dollar can be explained as follows. While both nominal prices fall in the transition from state 1 to state 2, as supply increases in both countries, the Chinese good becomes relatively cheaper due to China's higher output growth. Hence, the Chinese currency has a higher rate of return. Because the current generation expects that the current state realizes tomorrow with a high probability, this generates a higher demand for the Chinese currency. Hence, currency 2 (yuan) has to appreciate in equilibrium.

In Table 3, we report the share of world GDP and the money holdings of the two countries in the two states of nature.

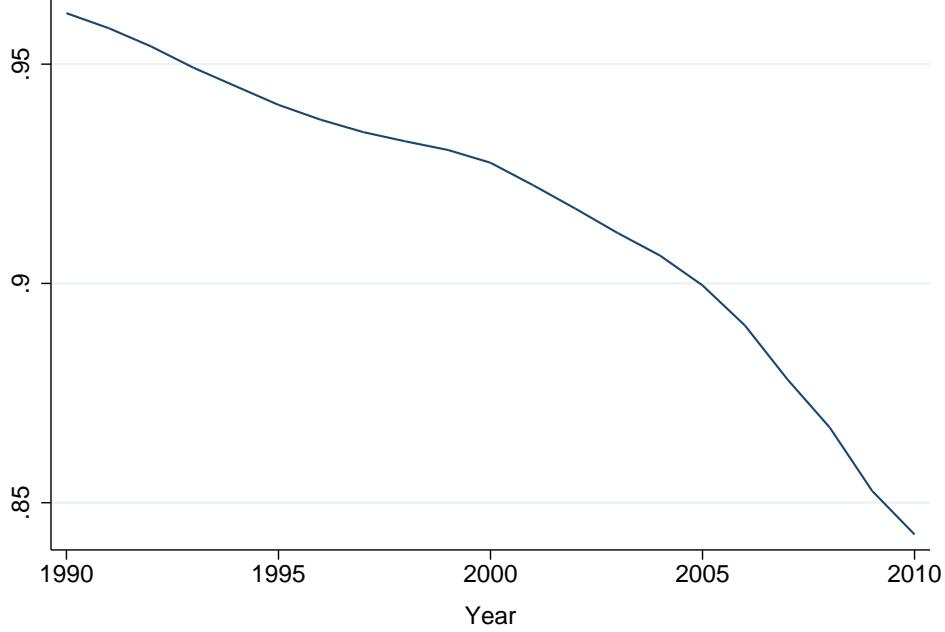
Since China's real GDP growth rate is higher than the US, its share of world GDP increases and therefore the young born in 2010 accumulate more assets than the previous generation. Despite the catching-up, China is still poorer than the US and holds a lower share of both the domestic and the foreign currency. If there was no home bias, the share of the assets held by each country would simply be equal to its share of world GDP according to Proposition 1. Since we have now allowed for the possibility of home bias,

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<sup>35</sup>We thank Michael Reiter for this nice interpretation.

this equivalence does not hold precisely. The consequence of home bias in consumption is that each country is holding more domestic than foreign assets in any given period.

Figure 5: The US share of “World GDP”, 1990-2010



Note. World GDP is calculated using gross domestic product per capita based on PPP per capita GDP at current international dollar. IMF data.

The model predicts a drop of the US share of world GDP by around 10%, and this is consistent with the US data as shown by Figure 5. Since this is a two-country model, the share of world GDP is simply the ratio of the US gross domestic product divided by the sum of the GDPs of China and the US<sup>36</sup>.

<sup>36</sup>In this model, we do not consider the potential impact of differential population growth rates on GDP and we look at GDP per capita for both countries, where GDP growth only comes from an increase in real GDP. This is not a restrictive assumption in this framework, as the average growth rate of the working population in the US and China is quite similar over the twenty-years period (e.g. see Eugeni, 2015).

Table 3: The share of “world GDP” and currency holdings. Baseline model.

		1990	2010
US	Share of “world GDP”	$\frac{w_1(1)}{\sum_h w_h(1)} = 0.8966$	$\frac{w_1(2)}{\sum_h w_h(2)} = 0.8$
	Share of domestic currency held	$\frac{m_1^1(1)}{M^1} = 0.9249$	$\frac{m_1^1(2)}{M^1} = 0.8727$
	Share of foreign currency held	$\frac{m_1^2(1)}{M^2} = 0.6508$	$\frac{m_1^2(2)}{M^2} = 0.5091$
China	Share of “world GDP”	$\frac{w_2(1)}{\sum_h w_h(1)} = 0.1034$	$\frac{w_2(2)}{\sum_h w_h(2)} = 0.2$
	Share of domestic currency held	$\frac{m_2^2(1)}{M^2} = 0.3492$	$\frac{m_2^2(2)}{M^2} = 0.4909$
	Share of foreign currency held	$\frac{m_2^1(1)}{M^1} = 0.0751$	$\frac{m_2^1(2)}{M^1} = 0.1273$

Using equations (28) and (32), we can compute the balance of trade, the change in the net foreign assets position and valuation effects for the US. The key identity that pins down the various components of the net external position of China can be derived following the same steps as for country 1. We report our variables as percentages of the respective country's GDP in Table 4.

Table 4: The net external positions of the US and China, 2010. Baseline model.

	Trade balance % GDP	Valuation effects % GDP	Change in NFA % GDP
US	$tb_1(12) = -4.1073$	$val_1(12) = 2.9559$	$\Delta nfa_1(12) = -1.1514$
China	$tb_2(12) = 16.4316$	$val_2(12) = -11.8252$	$\Delta nfa_2(12) = 4.6063$

Table 5: The net external positions of the US and China, 1990-2010. Data.

	$\frac{\sum_{t=1990}^{2010} CA_t}{GDP_{2010}} \%$	$\frac{\sum_{t=1990}^{2010} VAL_t}{GDP_{2010}} \%$	$\frac{NFA_{2010} - NFA_{1990}}{GDP_{2010}} \%$
United States	-41%	26%	-15%
China	31%	-6%	25%
United States vs. China	-15%		

*Notes.* The change in the net foreign assets position is calculated using the database of Lane and Milesi-Ferretti (2007). The current account of the US as well as the US position against China is calculated using data from the Bureau of Economic Analysis although the second time-series only starts from 1999. Valuation effects are computed as the difference between the change in the net foreign assets' position and the current account.

As the US experience lower growth, they run a (cumulated) trade deficit of more than 4% of domestic GDP. The asset side explanation is that, as the Chinese become wealthier, they accumulate more domestic and foreign assets. Hence, the US accumulate less foreign assets and hold more foreign liabilities and their net foreign assets position worsens. In an overlapping-generations model, the trade balance is backward-looking as well as forward-looking since the aggregate consumption of the country is composed of the consumption of the old as well as the young. Therefore, the country that runs a surplus is not the richest country in the current state, but the country whose relative position in the world economy has improved over time. In the goods' markets, the US import more and export less than before since Chinese goods become relatively cheaper<sup>37</sup>.

The trade deficit is partially offset by substantial positive valuation effects. As the

<sup>37</sup>Plugging the budget constraints into (28), we can write the other definition of the balance of trade as the difference between exports and imports:  $tb_1(s's) \equiv p^1(s)[c_{12}^1(s) + c_{22}^1(s's)] - p^2(s)e(s)[c_{11}^2(s) + c_{21}^2(s's)]$

Chinese currency appreciates, the value of the US foreign assets increases relatively to the value of foreign liabilities. Therefore, the US experience a positive wealth effect that mitigates the negative impact of the trade deficit on the external position of the country. On the other hand, China experiences a negative wealth effect as the country runs a trade surplus.

To compare our results with the data, we calculate the change in the net foreign assets positions, the current account and valuation effects of the US and China as accumulated over the past 20 years as a percentage of GDP in 2010.

First of all, Table 5 shows that the US current account deficit against China is one of the main driving forces behind the US current account deficit. Our model can explain almost a third of the US-China current account imbalance (4% out of 15%). Moreover, the model can capture the fact that valuation effects have stabilized both the net external positions of China and the United States. While the Bureau of Economic Analysis provides the current account position of the US disaggregated by country, we do not possess data on the US net foreign assets position versus China. As a consequence, we cannot compute the valuation effects between the US and China as a residual and make a full comparison between the model and the data. Yet, our result that valuation effects substantially contribute to the dynamics of the net foreign assets positions of the two countries is consistent with the overall data for the US and China.

The attentive reader will have noticed that the model overestimates the Chinese valuation effects with respect to the data. The reason is that we make the assumption that the Chinese exchange rate regime is freely floating, while the bulk of the Chinese currency appreciation happened after the reforms that took place in the mid-2000s. It is important to stress that our model can only capture low-frequency movements of the exchange rate and the balance of trade and does not aim at explaining high-frequency movements (or lack of) in foreign exchange markets. The model predicts that country 2's currency appreciates by 61% while the renmbimbi has only appreciated by 25% with respect to the dollar over the period 1994-2010<sup>38</sup>. This indicates that the wealth transfer from China to the US would have been much bigger if the Chinese had a fully floating nominal exchange rate regime.

The result that exchange rate-driven valuation effects are stabilizing is consistent with the empirical literature, which has found that the correlation between the trade balance

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<sup>38</sup>We use the end-of-the-period exchange rate time-series provided by Lane and Milesi-Ferretti (2007).

and valuation effects is negative (see e.g. Lane et al., 2002), and with the findings of other theoretical papers, which have shown that asset price-driven valuation effects are also stabilizing (e.g. Pavlova et al., 2010; Devereux et al., 2010). As Gourinchas et al. (2007) and Lane et al. (2010) have shown, exchange rate-driven valuation effects are also empirically relevant and this paper shows that one possible mechanism behind them is differences in output growth rates across countries. In particular, our mechanism provides an explanation behind the stylized facts presented in Figures 1-4, which show that the US have experienced positive valuation effects over the past 20 years despite accumulating a substantial trade deficit, while many emerging economies have experienced exactly the reverse, as a result of the increase in the share of world GDP of emerging countries (and especially China).

## 4.2 The importance of the valuation channel in the dynamics of net foreign assets

The other relevant contribution of this paper is that the model can generate meaningful *expected* valuation effects, which substantially contribute to the dynamics of net foreign assets.

Following the analysis in section 2.4, we can derive a forward-looking expression for the net foreign assets position of the US in 1990.

$$NFA_1(1) = E_{1s'}NFA_1(s') - E_{1s'}TB_1(1s') - E_{1s'}VAL_1(1s') \quad (41)$$

where  $E_{1s'}$  is the expectation operator which is conditional on the fact that the world economy is in state 1 in 1990.

Using our definitions (32), (33) and (35), the expected valuation effects can be calculated as follows:

$$\begin{aligned} E_{1s'}VAL_1(1s') &\equiv FA_1(1)E_{1s'}r(1s') = \\ &= FA_1(1)[\rho(11)r(11) + \rho(12)r(12)] = \rho(12)VAL_1(12) \end{aligned} \quad (42)$$

since  $r(11) = 0$ . Equation (42) shows that as long as the nominal exchange rate fluctuates across the two states of nature ( $r(12) \neq 0$ ), then expected valuation effects are different from zero.

Applying equation (36), we can calculate the unexpected component of valuation effects



as follows:

$$UNVAL_1(12) = VAL_1(12) - \rho(12)VAL_1(12) = \rho(11)VAL_1(12) \quad (43)$$

Since we have set  $\rho(ss) = 0.9$ , 90% of the observed valuation effects for the US between 1990 and 2010 are unexpected and the remaining 10% expected. Hence, equation (43) seems to indicate that the higher is the persistence of output the bigger is the unexpected component of valuation effects. Nonetheless, we will show in the sensitivity analysis that the importance of the expected valuation component in the dynamics of external adjustment actually increases with  $\rho(ss)$  as the realized valuation effects are relatively larger.

Table 6 reports the percentage of the net foreign assets position explained by each component in (41).

Table 6: A decomposition of the net external positions of the US in 1990

$\frac{E_{1s'}NFA_1(s')}{NFA_1(1)}\%$	—	$\frac{E_{1s'}TB_1(1s')}{NFA_1(1)}\%$	—	$\frac{E_{1s'}VAL_1(1s')}{NFA_1(1)}\%$
82.05	—	−64.03	—	46.08

The table shows that all three components have been important in driving the adjustment of the net external position of the US. Since growth was expected to be higher in China, then the model predicts that we should have expected a process of adjustment of the net external position of the US driven by a deterioration of the net foreign assets' position due to a trade deficit as well as positive valuation effects.

As  $NFA_1(1) > 0$ , the net external position remains (changes to) positive (negative) with higher (lower) probability as output is persistent. A great percentage of  $NFA$  is explained by the expected  $NFA$  position, which is of the same sign as the actual position. However, agents expect that the  $NFA$  will be lower than the current  $NFA$  as they expect a trade deficit. If the same state realizes next period, then the trade balance will be zero (equation (28)) and agents would regard this as the most likely outcome. However, there is a positive probability in 1990 that the distribution of wealth would shift in favor of China leading to a trade deficit, hence agents would expect a trade deficit overall. As the expected trade balance component has a negative sign, then the expected stock and flow components of net foreign assets would more than explain the current  $NFA$  position. But finally, and most importantly, agents would expect an exchange rate depreciation due to

the higher growth in China, hence the positive expected valuation effects.

Table 6 shows that a fundamental part of the process of the external adjustment between 1990 and 2010 can be explained by expected valuation effects. The literature on valuation effects to date has been able to generate large unexpected valuation effects (see e.g. Nguyen, 2011), but the dynamics of net foreign assets is still driven by the trade balance channel as shown by Tille et al. (2010), Devereux et al. (2010) and Ghironi et al. (2015).

Suppose that we wanted to use our framework to predict the process of adjustment of the present net foreign assets position of the US. In this case, the signs of the various components would be exactly the opposite<sup>39</sup>:

$$\underbrace{NFA_1(2)}_{(-)} = \underbrace{E_{2s'}NFA_1(s')}_{(-)} - \underbrace{E_{2s'}TB_1(2s')}_{(+)} - \underbrace{E_{2s'}VAL_1(2s')}_{(-)} \quad (44)$$

In particular, we should expect that the net foreign assets position improves thanks to a trade surplus and but we should also expect negative valuation effects for the US. This might seem in contradiction with Gourinchas et al. (2007), who find that future trade surpluses for the US should be accompanied by positive valuation effects instead. As we explained in section 2.4, our equation is more general than Gourinchas et al. (2007) since they assume that the no-Ponzi game condition holds, which implies that  $E_{2s'}NFA_1(s') = 0$ . Hence, we cannot expect to fully relate our results with their empirical analysis. It is easy to see that the restriction that  $E_{2s'}NFA_1(s') = 0$  automatically implies that  $E_{2s'}VAL_1(2s') > 0$  when  $E_{2s'}TB_1(2s') > 0$ , but that is not necessarily the case when  $E_{2s'}NFA_1(s') \neq 0$ . Yet, our framework supports their empirical finding that expected valuation effects are a fundamental component of the process of external adjustment by suggesting a mechanism based on expected growth differentials across countries.

## 5 Sensitivity analysis

The aim of this section is to check the robustness of the baseline model to alternative specifications of  $\sigma$  and  $\rho(ss)$ . Finally, we also allow for the intertemporal elasticity of substitution to differ from the intratemporal elasticity of substitution. While the model loses analytical tractability, our results are not affected by introducing a more general specification.

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<sup>39</sup>This only holds under the assumption that the US are expected to grow more than China in the future.

## 5.1 The elasticity of substitution

Table 7 shows that valuation effects always act as a stabilizer of the net external positions of the two countries, independently from the chosen value of the elasticity of substitution. As we explained in the previous section, the most realistic values for the elasticity is between 4 and 15 considering that we deal with low-frequency data (see Ruhl (2010)). We do not show the results for  $0 < \sigma < 1$ , as this would corresponds to a situation of immiserizing growth. For completeness, we also report the case in which  $\sigma = 1$ , which corresponds to the log case whose analytical solution we derive in Appendix B.

As the elasticity of substitution increases, goods are increasingly substitutable and agents do not react as much to changes in prices in their demand for the currencies. Therefore, the nominal exchange rate has less of a “shock-absorbing” role and valuation effects become less important relatively to the trade imbalances. As a consequence, the change in the net foreign asset positions of the two countries increases with the elasticity. However, valuation effects still account for a significant proportion of the dynamics of net foreign assets.

Table 7: Varying the elasticity of substitution

	Trade balance % GDP	Val. effects % GDP	$\Delta NFA$ % GDP
<i>United States</i>			
$\sigma = 1$	0	0	0
$\sigma = 2$	-4.0856	3.4385	-0.6471
$\sigma = 4$	-4.1073	2.9559	-1.1514
$\sigma = 8$	-3.8822	2.1888	-1.6933
$\sigma = 10$	-3.8046	1.9322	-1.8724
$\sigma = 16$	-3.6449	1.4110	-2.2339
<i>China</i>			
$\sigma = 1$	0	0	0
$\sigma = 2$	11.2413	-9.4607	1.7806
$\sigma = 4$	16.4316	-11.8252	4.6063
$\sigma = 8$	18.2916	-10.3132	7.9784
$\sigma = 10$	18.4841	-9.3874	9.0967
$\sigma = 16$	18.5050	-7.1336	11.3415

## 5.2 The relationship between the persistence of output and valuation effects

Table 8 shows that, for valuation effects to stabilize the net external positions of a country, the probability that next period's state is the same as the current period's must be sufficiently high. Valuation effects are stabilizing as long as  $\rho(ss) \geq 0.5$  but move in the same direction as the country's trade balance when  $\rho(ss) = 0.4$ <sup>40</sup>. Therefore, there is some cut-off value of  $\rho(ss)$  between 0.4 and 0.5 below which valuation effects are no longer stabilizing.

When there is no home bias ( $a_h^1 = a_h^2$ ), it is easy to see that this cut-off value is exactly 0.5. When  $\rho(ss) = 0.5$ , agents born in different states attach the same probabilities to future states, which implies that the exchange rate is constant in a stationary equilibrium hence valuation effects are zero (see equations (40) and (32)). When there is home bias, this cut-off value is smaller therefore valuation effects are stabilizing for a larger set of values of  $\rho(ss)$ . The condition under which valuation effects are stabilizing is not very stringent, as it only requires that output is somewhat persistent in the two countries.

Table 8 also indicates that valuation effects contribute more to the change in net foreign

<sup>40</sup>In this case, state 1 is more likely to occur when state 2 is realized hence agents would expect more inflation in country 2. This would lead to more demand for currency 1, hence an appreciation and negative valuation effects for the US.

Table 8: Varying output persistence

	Trade balance % GDP	Val. effects % GDP	$\Delta NFA$ % GDP
<i>United States</i>			
$\rho(ss) = 0.4$	-2.9122	-0.0838	-2.9960
$\rho(ss) = 0.5$	-3.0204	0.1964	-2.8240
$\rho(ss) = 0.6$	-3.1670	0.5741	-2.5929
$\rho(ss) = 0.7$	-3.3715	1.0969	-2.2747
$\rho(ss) = 0.8$	-3.6663	1.8430	-1.8232
$\rho(ss) = 0.9$	-4.1073	2.9559	-1.1514
<i>China</i>			
$\rho(ss) = 0.4$	12.1836	0.3508	12.5344
$\rho(ss) = 0.5$	12.5771	-0.8178	11.7593
$\rho(ss) = 0.6$	13.1065	-2.3757	10.7308
$\rho(ss) = 0.7$	13.8393	-4.5024	9.3370
$\rho(ss) = 0.8$	14.8850	-7.4827	7.4022
$\rho(ss) = 0.9$	16.4316	-11.8252	4.6063

assets the higher is the persistence of output. The reason is that the exchange rate is more volatile, as the expectations of individuals born in different states of nature vary more widely.

In Table 9, we explore how the various components of adjustment in the dynamics of the net external position of the US vary with  $\rho(ss)$ .

Table 9: A decomposition of the net external positions of the US in 1990 for different values of  $\rho(ss)$

	$\frac{E_{1s'} NFA_1(s')}{NFA_1(1)} \%$	—	$\frac{E_{1s'} TB_1(1s')}{NFA_1(1)} \%$	—	$\frac{E_{1s'} VAL_1(1s')}{NFA_1(1)} \%$
$\rho(ss) = 0.5$	−24.44	—	−133.099	—	8.65
$\rho(ss) = 0.6$	5.70	—	−115.18	—	20.88
$\rho(ss) = 0.7$	33.96	—	−97.89	—	31.85
$\rho(ss) = 0.8$	59.69	—	−81.06	—	40.75
$\rho(ss) = 0.9$	82.05	—	−64.03	—	46.08

As  $\rho(ss)$  increases, the valuation channel becomes increasingly important in explaining the process of adjustment of net foreign assets' position. Although the expected component is relatively smaller than the unexpected component (equations (42) and (43)), the realized valuation effects are quantitatively larger (Table 8) hence they can explain a significant percentage of the net external position.

To conclude, we have shown that the relative importance of the valuation channel as opposed to the trade balance channel crucially depends on the nature of the stochastic process of output: when there is high persistence in the economy, the nominal exchange rate is more volatile and the valuation component becomes a crucial adjustment mechanism.

### 5.3 Separating the intertemporal and the intratemporal elasticities of substitution

In this section, we relax Assumption 1 to take into account that the intertemporal elasticity of substitution is typically much lower than the elasticity of substitution between traded goods. While the baseline model was partially tractable, this generalized version is not and finding its solution involves the numerical computation of a large system of nonlinear equations, which includes the first-order conditions of the agents, the budget constraints as well as the market clearing equations. Even in our simple two-states exam-

ple, the equilibrium system is quite large as it consists of 22 equations and unknowns<sup>41</sup>. We show in detail how to solve this generalized version of the model in Appendix C.

For our sensitivity analysis, we choose the same coefficient of risk aversion as Corsetti et al. (2008)<sup>42</sup>. In this section, we consider a value for the discount factor which is standard in the real business cycle literature. Since we assume that a period lasts 20 years, then a quarterly discount rate of 0.99 implies that  $\beta = 0.99^{20 \times 4} = 0.45$  (see also De La Croix and Doepke, 2003; Gottardi and Kubler, 2011). The other parameters are the same as in the baseline model.

Table 10: Parameter Values. Generalized model.

Coefficient of risk aversion	$\gamma_1 = \gamma_2 = 2$
Discount factor	$\beta_1 = \beta_2 = 0.45$

As the following table shows, the quantitative results of this more general version of the model is not very different from the baseline.

Table 11: The net external positions of the US and China, 2010. A more general model.

	Trade balance % GDP	Valuation effects % GDP	Change in NFA % GDP
US	$tb_1(12) = -4.2796$	$val_1(12) = 2.4244$	$\Delta nfa_1(12) = -1.8552$
China	$tb_2(12) = 17.3641$	$val_2(12) = -9.8367$	$\Delta nfa_2(12) = 7.5274$

## 6 Conclusions

This paper presents a two-country model of nominal exchange rate determination where exchange rate-driven valuation effects substantially contribute to the dynamics of the net foreign assets' position of a country. If real output growth rates differ across countries, the distribution of world GDP varies over time and trade imbalances among the two countries arise. In particular, a country runs a trade surplus if its relative position in the world economy improves, as measured by the share of world GDP. As the current generation is wealthier than the previous one, it holds a higher share of the available assets in the world economy and therefore the country's net foreign assets increase. We provide evidence to

<sup>41</sup>A higher number of countries or states of nature would substantially increase the dimension of the system.

<sup>42</sup>In Appendix C, we show that our results are not particularly sensitive to the degree of risk aversion.

show that the mechanism which generates trade imbalances in the model is empirically relevant. The nominal exchange rate fluctuates according to agents' expectations about the relative purchasing power of the two currencies: if a currency is expected to have a higher purchasing power in the future (because of higher output growth in the home country), then an increase in demand for that currency causes an exchange rate appreciation. Exchange rate fluctuations generate quantitatively big valuation effects for reasonable parameter values, which stabilize the countries' net foreign assets positions. Our model is consistent with the stylized facts on the net external positions that the United States and East Asian countries have built up over the past twenty years and the depreciation of the dollar against the currencies of emerging market economies. This model also shows that expected valuation effects are an important part of the process of external adjustment of a country, as the empirical literature has emphasized (see Gourinchas et al., 2007).

One of the main challenges ahead is to design a model that can simultaneously explain exchange rate and asset price-driven valuation effects. This would be important in order to understand whether or under which conditions these two sources of valuation effects behave differently from a qualitative point of view, as well as their relative importance from a quantitative point of view. Another important issue which we leave to future research is the investigation of the role of monetary policy in a world where exchange rate fluctuations affect the net external positions of many countries. Do central banks amplify valuation effects while pursuing their monetary policies? But even more importantly, should central banks be worried about these huge wealth effects or is exchange rate volatility harmless instead? These are very open questions and beyond the scope of this paper. Our framework can explain some relevant trends in international financial markets and it is a promising line of research in the direction of answering other important research questions.

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## 7 Appendix

### 7.1 Appendix A

#### 7.1.1 The (in)determinacy of the nominal exchange rate and portfolios

In this section, we clarify why the requirement that agents must choose their portfolio of currencies before uncertainty is realized (without being able to readjust their decision) is crucial to guarantee exchange rate and portfolio determinacy.

Let us now assume that agents can adjust their portfolio decision once the value of the shock is known. The budget constraints of the old would be written as follows:

$$\begin{aligned}
 m_h^1(ss') + e(s')m_h^2(ss') &= m_h^1(s) + e(s')m_h^2(s) \\
 p^1(s')c_{2h}^1(ss') &= m_h^1(ss') \\
 p^2(s')c_{2h}^2(ss') &= m_h^2(ss')
 \end{aligned}$$

The main implication is that the old would be allowed to trade the currencies in the foreign exchange markets before the goods' markets open. After they adjust their portfolios, they would spend their currencies in the respective markets according to their cash-in-advance constraints. However, the above constraints can be consolidated as follows:

$$p^1(s')c_{2h}^1(ss') + p^2(s')e(s')c_{2h}^2(ss') = m_h^1(s) + e(s')m_h^2(s) \quad (45)$$

Under this structure, our economy would be practically identical to Manuelli et al. (1990), where the nominal exchange rate and portfolios are indeterminate. In the deterministic world of Kareken et al. (1981), the two currencies are perfect substitutes as stores of value, hence a portfolio of currencies cannot be pinned down. The nominal exchange rate is constant in equilibrium and its value cannot be determined. Manuelli et al. (1990) find the same indeterminacy result in a stochastic OLG model. Although the nominal exchange rate can fluctuate, the equilibrium path for the nominal exchange rate is still indeterminate. The only difference between our framework and theirs is that ours is a two-good economy while theirs is a one-good economy. However, the indeterminacy of the nominal exchange rate is not solved by augmenting the number of goods.

### 7.1.2 Derivation of portfolios

In this section, we explain how to derive the demand for the currencies.

First, combine (15), (12) and (14) for  $\ell = 1$ :

$$\frac{c_{1h}^1(s)^{-\frac{1}{\sigma}}}{p^1(s)} = \beta \sum_{s'} \frac{\rho(ss')c_{1h}^1(ss')^{-\frac{1}{\sigma}}}{p^1(s')} \quad (46)$$

and rewrite it as follows:

$$\frac{p^1(s)^{\frac{1-\sigma}{\sigma}}}{[p^1(s)c_{1h}^1(s)]^{\frac{1}{\sigma}}} = \beta \sum_{s'} \frac{\rho(ss')p^1(s')^{\frac{1-\sigma}{\sigma}}}{[p^1(s')c_{1h}^1(ss')]^{\frac{1}{\sigma}}} \quad (47)$$

Plugging  $p^1(s')c_{2h}^1(ss') = m_h^1(s)$  for every  $s'$ , we can sum up the numerators in the right hand side and elevate both sides of the equation to  $\sigma$ :

$$\frac{p^1(s)^{1-\sigma}}{p^1(s)c_{1h}^1(s)} = \beta^\sigma \frac{\left[ \sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{m_h^1(s)} \quad (48)$$

Next, we combine the first-order conditions for the goods consumed when young as follows:

$$\frac{a_h^{\frac{1}{\sigma}}c_{1h}^1(s)^{-\frac{1}{\sigma}}}{p^1(s)} = \frac{a_h^{\frac{2}{\sigma}}c_{1h}^2(s)^{-\frac{1}{\sigma}}}{p^2(s)e(s)} \quad (49)$$

After some manipulations, the above equation can be rewritten as follows:

$$\frac{p^2(s)e(s)c_{1h}^2(s)}{a_h^2} = \frac{[p^2(s)e(s)]^{1-\sigma}}{p^1(s)^{1-\sigma}} \frac{p^1(s)c_{1h}^1(s)}{a_h^1} \quad (50)$$

Now, plug (50) into the budget constraint when young and obtain:

$$p^1(s)c_{1h}^1(s) = \frac{a_h^1 p^1(s)^{1-\sigma}}{a_h^1 p^1(s)^{1-\sigma} + a_h^2 [p^2(s)e(s)]^{1-\sigma}} [w_h(s) - m_h^1(s) - e(s)m_h^2(s)] \quad (51)$$

Plug it into (48) and rearrange:

$$m_h^1(s) = \frac{a_h^1 \beta^\sigma \left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma [w_h(s) - e(s)m_h^2(s)]}{a_h^1 p^1(s)^{1-\sigma} + a_h^2 [p^2(s)e(s)]^{1-\sigma} + a_h^1 \beta^\sigma \left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma} \quad (52)$$

Now, combine (16) with (14) for  $\ell = 2$ :

$$\lambda_h(s)e(s) = a_h^2 \beta \sum_{s'} \frac{\rho(ss') c_{2h}^2(ss')^{-\frac{1}{\sigma}}}{p^2(s')} \quad (53)$$

Multiplying and dividing each term of the right hand side by  $p^2(s')^{\frac{1}{\sigma}}$  and then substituting  $p^2(s')c_{2h}^2(ss') = m_h^2(s)$ , we can sum the numerators on the right hand side and get the following equation:

$$\lambda_h(s) = a_h^2 \beta \frac{\sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}}}{m_h^2(s)^{\frac{1}{\sigma}} e(s)} \quad (54)$$

Because  $\lambda_h(s) = \sum_{s'} \lambda_h^1(ss')$ , we can write:

$$\underbrace{\frac{a_h^1 \beta \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}}}{m_h^1(s)^{\frac{1}{\sigma}}}}_{\sum_{s'} \lambda_h^1(ss')} = a_h^2 \beta \frac{\sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}}}{m_h^2(s)^{\frac{1}{\sigma}} e(s)} \quad (55)$$

or

$$\frac{m_h^1(s)}{m_h^2(s)} = e(s)^\sigma \frac{a_h^1}{a_h^2} e(s)^\sigma \frac{\left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{\left[ \sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma} \quad (56)$$

Solving (56) and (52) simultaneously, we obtain the demand for the two currencies:

$$m_h^1(s) = \frac{a_h^1 \beta^\sigma \left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{A_h(s)} w_h(s) \quad (57)$$

$$m_h^2(s) = \frac{a_h^2 \beta^\sigma e(s)^{-\sigma} \left[ \sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma}{A_h(s)} w_h(s) \quad (58)$$

where

$$\begin{aligned} A_h(s) &\equiv a_h^1 p^1(s)^{1-\sigma} + a_h^2 [p^2(s)e(s)]^{1-\sigma} + a_h^1 \beta^\sigma \left[ \sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma + \\ &+ a_h^2 \beta^\sigma e(s)^{1-\sigma} \left[ \sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}} \right]^\sigma \end{aligned}$$

### 7.1.3 Derivation of the demand functions of the young

Let us recall the budget constraint of agent  $h$  born in state  $s$ :

$$p^1(s)c_{1h}^1(s) + p^2(s)e(s)c_{1h}^2(s) = w_h(s) - m_h^1(s) - e(s)m_h^2(s)$$

Firstly, we can obtain total expenditure by substituting the demand for the currencies (20) and (21):

$$p^1(s)c_{1h}^1(s) + p^2(s)e(s)c_{1h}^2(s) = \frac{a_h^1 p^1(s)^{1-\sigma} + a_h^2 [p^2(s)e(s)]^{1-\sigma}}{A_h(s)} w_h(s) \quad (59)$$

Combining the above equation with (50), we can derive the demand functions of the young agents.

## 7.2 Appendix B. Solution of the model

### 7.2.1 Methodology and numerical solution

First, the number of equations in the goods' markets can be reduced by applying Walras Law. Suppose that money markets clear. If we sum across agents the budget constraints of the young and the old and combine them, we get the following equation:

$$p^1(s)[c_1^1(s) + c_2^1(s's) - y^1(s)] + p^2(s)e(s)[c_1^2(s) + c_2^2(s's) - y^2(s)] = 0 \quad \forall s', s$$

For every pair of  $(s', s)$ , if the market for good 1 clears then the market for good 2 clears automatically. Therefore,  $S^2$  equations can be made redundant.

Given Walras Law, suppose that the independent equations in the goods' markets are those for good 1. The next step is to sum across agents the budget constraints of the old for good 1 in state  $s$  so that we obtain the following equation:

$$p^1(s)c_2^1(s's) = M^1$$

It is easy to see that the aggregate consumption of the old does not depend on the previous state (the state realized when born) as aggregate real money balances only depend on the current state:

$$c_2^1(s's) = \frac{M^1}{p^1(s)} \quad \Rightarrow \quad c_2^1(s's) = c_2^1(s)$$

Given that the aggregate consumption of the old for good 1 does not depend on the past, it is enough that  $S$  equations in the market for good 1 clears. For instance, assume that the independent equations are those for which  $s' = s$ :

$$c_1^1(s) + c_2^1(ss) = y^1(s)$$

Therefore, the solution method involves to find the nominal prices and the exchange rates which solve the following system of  $3S$  equations:

$$\begin{aligned}\sum_h c_{1h}^1(s) + \sum_h c_{2h}^1(ss) &= y^1(s) \\ \sum_h m_h^1(s) &= M^1 \\ \sum_h m_h^2(s) &= M^2\end{aligned}$$

In section 4, we assume that  $S = 2$  which implies that we have a system of 6 equations and unknowns to solve. After plugging the demand functions for goods and money balances into the above market clearing equations, we solve the system numerically using Matlab.

The solution under the parameter values chosen in section 4 is:

$$\begin{aligned}p^1(1) &= 6.1486 \times 10^{-5} & p^1(2) &= 4.9539 \times 10^{-5} \\ p^2(1) &= 8.1947 \times 10^{-4} & p^2(2) &= 2.9220 \times 10^{-4} \\ e(1) &= 0.1357 & e(2) &= 0.2293\end{aligned}$$

It is also useful to compute relative prices, expressed in the numéraire currency, as follows:  $p(s) \equiv \frac{p^2(s)e(s)}{p^1(s)}$ . Therefore:

$$\begin{aligned}p(1) &= 1.8079 \\ p(2) &= 1.3526\end{aligned}$$

When  $p(s)$  falls, the price of US imports (exports) falls (increases). Therefore the US experiences an improvement of the terms of trade in the transition from state 1 to state 2.

### 7.2.2 An analytically tractable version: log utility

In this section, we derive the baseline model assuming that  $\sigma \rightarrow 1$ .

Agent  $h$  born in state  $s$  solves the following maximization problem:

$$\max \sum_{\ell} a_h^{\ell} \log c_{1h}^{\ell}(s) + \beta \sum_{s'} \rho(ss') \sum_{\ell} a_h^{\ell} \log c_{2h}^{\ell}(ss')$$

subject to (9), (10) and (11). The first-order conditions are:

$$c_{1h}^1(s) : \frac{a_h^1}{c_{1h}^1(s)} = \lambda_h(s)p^1(s) \quad (60)$$

$$c_{1h}^2(s) : \frac{a_h^2}{c_{1h}^2(s)} = \lambda_h(s)p^2(s)e(s) \quad (61)$$

$$c_{2h}^\ell(ss') : \frac{\beta a_h^\ell \rho(ss')}{c_{2h}^\ell(ss')} = \lambda_h^\ell(ss')p^\ell(s') \quad \forall \ell, s' \quad (62)$$

$$m_h^1(s) : -\lambda_h(s) + \sum_{s'} \lambda_h^1(ss') = 0 \quad (63)$$

$$m_h^2(s) : -\lambda_h(s)e(s) + \sum_{s'} \lambda_h^2(ss') = 0 \quad (64)$$

$$\begin{aligned} \lambda_h(s) : & p^1(s)[c_{1h}^1(s) - \omega_h^1(s)] + p^2(s)e(s)[c_{1h}^2(s) - \omega_h^2(s)] + \\ & + m_h^1(s) + e(s)m_h^2(s) = 0 \end{aligned} \quad (65)$$

$$\lambda_h^1(ss') : p^1(s')c_{2h}^1(ss') - m_h^1(s) = 0 \quad \forall s' \quad (66)$$

$$\lambda_h^2(ss') : p^2(s')c_{2h}^2(ss') - m_h^2(s) = 0 \quad \forall s' \quad (67)$$

Solving the above maximization problem involves the following steps. First, combine (60) and (61):

$$\frac{p^1(s)c_{1h}^1(s)}{a_h^1} = \frac{p^2(s)e(s)c_{1h}^2(s)}{a_h^2} \quad (68)$$

Plug the above into the young's budget constraint to obtain:

$$p^1(s)c_{1h}^1(s) = a_h^1[w_h(s) - m_h^1(s) - e(s)m_h^2(s)] \quad (69)$$

Take the first-order conditions for good 1 in all spots and plug them into (63):

$$\frac{a_h^1}{p^1(s)c_{1h}^1(s)} = \beta a_h^1 \sum_{s'} \frac{\rho(ss')}{p^1(s')c_{2h}^1(ss')} \quad (70)$$

Then, substitute (69) and (66) into (70) and obtain:

$$m_h^1(s) (1 + a_h^1 \beta) = \beta a_h^1 [w_h(s) - e(s)m_h^2(s)] \quad (71)$$

Now follow the same steps for good 2. First, take (68) and this time rewrite the budget constraint when young getting rid of good 1. Second, combine (64), (61) and (62) for good 2. Finally, plug in the rewritten budget constraint and (67):

$$e(s)m_h^2(s) (1 + a_h^2 \beta) = \beta a_h^2 [w_h(s) - m_h^1(s)] \quad (72)$$

Solve equations (71) and (72) simultaneously to obtain agent  $h$ 's demand for the curren-

cies:

$$\begin{aligned} m_h^1(s) &= \frac{\beta a_h^1}{1 + \beta} w_h(s) \\ m_h^2(s) &= \frac{\beta a_h^2}{1 + \beta} \frac{w_h(s)}{e(s)} \end{aligned}$$

The demand functions are:

$$\begin{aligned} c_{1h}^1(s) &= \frac{a_h^1}{1 + \beta} \frac{w_h(s)}{p^1(s)} \\ c_{1h}^2(s) &= \frac{a_h^2}{1 + \beta} \frac{w_h(s)}{p^2(s)e(s)} \\ c_{2h}^1(ss') &= \frac{\beta a_h^1}{1 + \beta} \frac{w_h(s)}{p^1(s')} \quad \forall s' \\ c_{2h}^2(ss') &= \frac{\beta a_h^2}{1 + \beta} \frac{w_h(s)}{p^2(s')e(s)} \quad \forall s' \end{aligned}$$

As in the more general case, demand is a linear functions of wealth. However, the demand for the goods and the two currencies are not functions of future prices and this makes the model analytically tractable.

In the main body of the paper, we have shown that only  $S$  equations in the goods' markets are independent. For instance, we can take the market clearing equations for good 1 when the previous state is equal to the current state:

$$\sum_h c_{1h}^1(s) + \sum_h c_{2h}^1(ss) = y^1(s)$$

Now, substitute the demand functions for good 1 into the market clearing equation:

$$\sum_h \frac{a_h^1}{1 + \beta} \frac{w_h(s)}{p^1(s)} + \sum_h \frac{\beta a_h^1}{1 + \beta} \frac{w_h(s)}{p^1(s)} = y^1(s)$$

Using the fact that  $w_1(s) = p^1(s)y^1(s)$  and  $w_2(s) = p^2(s)e(s)y^2(s)$ , the market clearing equation for good 1 pins down relative prices or the terms of trade in each state:

$$\frac{p^2(s)e(s)}{p^1(s)} = \frac{a_1^2 y^1(s)}{a_2^1 y^2(s)} \quad s = 1, \dots, S \quad (73)$$

Using (73), we can show that the money market clearing equations pin down nominal prices:

$$p^1(s) = \frac{M^1}{y^1(s)} \frac{1 + \beta}{\beta} \quad s = 1, \dots, S \quad (74)$$

$$p^2(s) = \frac{M^2}{y^2(s)} \frac{1 + \beta}{\beta} \quad s = 1, \dots, S \quad (75)$$



Finally, the exchange rate can be computed:

$$e(s) = e = \frac{M^1 a_1^2}{M^2 a_2^1} \quad (76)$$

Under log utility, it is remarkable that the exchange rate is constant even though the stochastic process is Markov and discount factors differ across agents.

Equations (74), (75) and (76) reveal that agents born in different states of nature have the same wealth. Therefore, it is immediate that the demand for the currencies are not state dependent:

$$m_h^\ell(s) = m_h^\ell \quad \forall h, \ell$$

As there is no portfolio rebalancing across states of nature, the balance of trade is always in equilibrium and the change in net foreign assets is equal to zero in all states (see section 4).

Finally, we can compute the consumption allocation:

$$\begin{aligned} c_{11}^1(s) &= \frac{a_1^1}{1+\beta} y^1(s) & c_{12}^1(s) &= \frac{a_1^2}{1+\beta} y^1(s) \\ c_{11}^2(s) &= \frac{a_2^1}{1+\beta} y^2(s) & c_{12}^2(s) &= \frac{a_2^2}{1+\beta} y^2(s) \\ c_{21}^1(s's) &= \frac{\beta a_1^1}{1+\beta} y^1(s) & c_{22}^1(s's) &= \frac{\beta a_1^2}{1+\beta} y^1(s) \\ c_{21}^2(s's) &= \frac{\beta a_2^1}{1+\beta} y^2(s) & c_{22}^2(s's) &= \frac{\beta a_2^2}{1+\beta} y^2(s) \end{aligned}$$

Note that since portfolio rebalancing does not occur, then the consumption of the old does not depend on the state of birth but simply on the state realized when old.

### 7.3 Appendix C. Separating the intertemporal and the intratemporal elasticities of substitution

To solve the agents' maximization problem, it is convenient to plug the budget constraints of the old (10) and (11) into the CES aggregators of the old (3) as follows:

$$c_{2h}(ss') \equiv \left[ a_h^{1/\sigma} \left( \frac{m_h^1(s)}{p^1(s')} \right)^{\frac{\sigma-1}{\sigma}} + a_h^{2/\sigma} \left( \frac{m_h^2(s)}{p^2(s')} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (77)$$

Therefore, our problem becomes to choose a portfolio of currencies  $m_h(s)$  and a consumption vector when young  $c_{1h}(s)$  which maximizes (1) subject to the CES aggregator of the young (2), the modified CES aggregator of the old (77) and the budget constraint of the young (9).

The first-order conditions are:

$$\begin{aligned}
c_{1h}^1(s) &: a_h^{\frac{1}{\sigma}} c_{1h}^1(s)^{-\frac{1}{\sigma}} \left[ a_h^{\frac{1}{\sigma}} c_{1h}^1(s)^{\frac{\sigma-1}{\sigma}} + a_h^{\frac{2}{\sigma}} c_{1h}^2(s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\gamma\sigma}{\sigma-1}} = \lambda_h(s) p^1(s) \\
c_{1h}^2(s) &: a_h^{\frac{2}{\sigma}} c_{1h}^2(s)^{-\frac{1}{\sigma}} \left[ a_h^{\frac{1}{\sigma}} c_{1h}^1(s)^{\frac{\sigma-1}{\sigma}} + a_h^{\frac{2}{\sigma}} c_{1h}^2(s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\gamma\sigma}{\sigma-1}} = \lambda_h(s) p^2(s) e(s) \\
m_h^1(s) &: \beta a_h^{\frac{1}{\sigma}} \sum_{s'} \left( \frac{m_h^1(s)}{p^1(s')} \right)^{-\frac{1}{\sigma}} \left[ a_h^{\frac{1}{\sigma}} \left( \frac{m_h^1(s)}{p^1(s')} \right)^{\frac{\sigma-1}{\sigma}} + a_h^{\frac{2}{\sigma}} \left( \frac{m_h^2(s)}{p^2(s')} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\gamma\sigma}{\sigma-1}} = \lambda_h(s) p^1(s') \\
m_h^2(s) &: \beta a_h^{\frac{2}{\sigma}} \sum_{s'} \left( \frac{m_h^2(s)}{p^2(s')} \right)^{-\frac{1}{\sigma}} \left[ a_h^{\frac{1}{\sigma}} \left( \frac{m_h^1(s)}{p^1(s')} \right)^{\frac{\sigma-1}{\sigma}} + a_h^{\frac{2}{\sigma}} \left( \frac{m_h^2(s)}{p^2(s')} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\gamma\sigma}{\sigma-1}} = \lambda_h(s) p^2(s') e(s) \\
\lambda_h(s) &: w_h(s) - m_h^1(s) - e(s) m_h^2(s) - p^1(s) c_{1h}^1(s) - p^2(s) e(s) c_{1h}^2(s) = 0
\end{aligned}$$

We can rewrite the above equations as follows:

$$\frac{a_h^{\frac{1}{\sigma}} c_{1h}^1(s)^{-\frac{1}{\sigma}}}{p^1(s)} = \frac{a_h^{\frac{2}{\sigma}} c_{1h}^2(s)^{-\frac{1}{\sigma}}}{p^2(s) e(s)} \quad (78)$$

$$\begin{aligned}
&\frac{c_{1h}^1(s)^{-\frac{1}{\sigma}}}{p^1(s)} \left[ a_h^{\frac{1}{\sigma}} c_{1h}^1(s)^{\frac{\sigma-1}{\sigma}} + a_h^{\frac{2}{\sigma}} c_{1h}^2(s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\gamma\sigma}{\sigma-1}} = \\
&= \beta \sum_{s'} \left( \frac{m_h^1(s)}{p^1(s')} \right)^{-\frac{1}{\sigma}} \frac{1}{p^1(s')} \left[ a_h^{\frac{1}{\sigma}} \left( \frac{m_h^1(s)}{p^1(s')} \right)^{\frac{\sigma-1}{\sigma}} + a_h^{\frac{2}{\sigma}} \left( \frac{m_h^2(s)}{p^2(s')} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\gamma\sigma}{\sigma-1}} \quad (79)
\end{aligned}$$

$$\begin{aligned}
&\frac{c_{1h}^2(s)^{-\frac{1}{\sigma}}}{p^2(s) e(s)} \left[ a_h^{\frac{1}{\sigma}} c_{1h}^1(s)^{\frac{\sigma-1}{\sigma}} + a_h^{\frac{2}{\sigma}} c_{1h}^2(s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\gamma\sigma}{\sigma-1}} = \\
&= \beta \sum_{s'} \left( \frac{m_h^2(s)}{p^2(s')} \right)^{-\frac{1}{\sigma}} \frac{1}{p^2(s')} \left[ a_h^{\frac{1}{\sigma}} \left( \frac{m_h^1(s)}{p^1(s')} \right)^{\frac{\sigma-1}{\sigma}} + a_h^{\frac{2}{\sigma}} \left( \frac{m_h^2(s)}{p^2(s')} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\gamma\sigma}{\sigma-1}} \quad (80)
\end{aligned}$$

$$m_h^1(s) + e(s) m_h^2(s) + p^1(s) c_{1h}^1(s) + p^2(s) e(s) c_{1h}^2(s) = w_h(s) \quad (81)$$

The first equation shows that the marginal rate of substitution between the two goods when young depends on the degree of home bias and the elasticity of substitution between the two goods. An identical equation does hold in the baseline model. On the other hand, equations (79) and (80) show the relationship between the marginal utility of consumption today and the marginal utility of consumption tomorrow for the respective goods. These also depend on the degree of the agent's risk aversion<sup>43</sup>.

Finally, we can derive a modified version of Walras Law by aggregating the budget constraints of the young (81) and assuming that the money markets clear:

$$p^1(s) \left( \sum_h c_{1h}^1(s) - y^1(s) \right) + p^2(s) e(s) \left( \sum_h c_{1h}^2(s) - y^2(s) \right) = -M^1 - e(s) M^2$$

<sup>43</sup>Notice that, if we impose  $\gamma\sigma = 1$  in the above equations, we retrieve the first-order conditions of the baseline model.

As long as the young's excess supply for good 1 is equal to the real money balances of currency 1 in each state

$$y^1(s) - \sum_h c_{1h}^1(s) = \frac{M^1}{p^1(s)}$$

the excess supply equation for good 2 clears automatically.

In this generalized version of the model, the utility function is not separable across goods and therefore we cannot derive analytically the demand functions as in the baseline model. Our equilibrium system is much larger than before and it is larger the higher is number of states of nature:

Table 12: Equilibrium system

<i>No. equations</i>	<i>No. unknowns</i>
$4 \times 2 \times S$ (consolidated) F.O.C.	$2 \times 2 \times S$ consumption allocations
$2 \times S$ money equations	$2 \times 2 \times S$ money holdings
$1 \times S$ real money balances	$2 \times S + 1 \times S$ nominal prices + exchange rates

4 consolidated first-order conditions need to hold for each agent, which have to be multiplied by the number of agents in the world economy and the states of nature. To those, it corresponds an equivalent number of unknowns in terms of consumption allocations of the two goods when young and the money holdings. Finally, the money markets' clearing equations and the equation for the real money balances for good 1 are matched by an equal number of prices. If we analyze a two-states system as above, then our equilibrium system consists of 22 equations and unknowns.

### 7.3.1 Robustness: alternative values of the risk aversion coefficient

The following table shows that the performance of the model is not particularly affected if we chose alternative values for the coefficient of risk aversion.

Table 13: Varying the coefficient of risk aversion: the net external position of the US

	Trade balance % GDP	Valuation effects % GDP	$\Delta NFA$ % GDP
$\gamma = 1$	-3.2456	1.9195	-1.3261
$\gamma = 2$	-4.2796	2.4244	-1.8552
$\gamma = 3$	-4.6995	2.2801	-2.4194
$\gamma = 4$	-4.9746	1.8951	-3.0795
$\gamma = 5$	-5.1945	1.4524	-3.7421